Ethica More Logico Demonstrata

Justin Bledin and Yitzhak Y. Melamed

October 8, 2019

1 Introduction

Returning to London in August 1661 after meeting Spinoza at his retreat in Rijnsburg, Henry Oldenburg, Secretary of the Royal Society, began corresponding with his new friend by asking him to elaborate on his views of God, extension, infinite thought, and a variety of other topics (Ep. 1). In his reply the following month (Ep. 2), we find Spinoza working out ideas that will later turn into the beginning of his magnum opus Ethica More Geometrico Demonstrata [Ethics, Demonstrated in the Geometrical Manner]. Spinoza’s letter opens with definitions of God and attribute (precursors to his definitions of God (E1d6) and substance (E1d3) in the Ethics) and then lists a series of principles expressing some of the basic properties of substance (ancestors to propositions E1p5, E1p6c, E1p7, and E1p8). Derivations of these principles in the “Geometric manner” were provided in a separate attachment to the letter, which was intended to bring Oldenburg to the truth as quickly and efficiently as possible:

---

1. All quotations from Spinoza’s works and letters are from Curley’s translation. We rely on Gebhardt’s critical edition (Spinoza Opera, four volumes (Heidelberg: Carl Winter Verlag, 1925)) for the Latin text. We use the following abbreviations for Spinoza’s cited works: ‘TdIE’ for Treatise on the Emendation of the Intellect [Tractatus de Intellectus Emendatione], ‘DPP’ for Descartes’ Principles of Philosophy [Renati des Cartes Principiorum Philosophiae Pars I & II], ‘CM’ for Metaphysical Thoughts [Cogitata Metaphysica], ‘KV’ for Short Treatise on God, Man, and His Well-Being [Korte Verhandeling van God de Mensch en deszelfs Welstand], and ‘Ep.’ for Letters. Passages in the Ethics are referred to using the following now-standard abbreviations: ‘a’ for axiom, ‘c’ for corollary, ‘e’ for explanation, ‘p’ for proposition, ‘s’ for scholium, and ‘d’ for a definition when it appears immediately to the right of the part of the book or a demonstration in all other cases (so, for example, E1d3 is the third definition of Part One of the Ethics, and E1p16d is the demonstration of the sixteenth proposition of Part One).
Once I have demonstrated these things, then (provided you attend to the definition of God), you will easily be able to see what I am aiming at, so it is not necessary to speak more openly about these matters. But I can think of no better way of demonstrating these things clearly and briefly than to prove them in the Geometric manner \textit{[more Geometrico]} and subject them to your understanding. So I send them separately with this letter and await your judgment regarding them. (Ep. 2)

Unfortunately, the enclosed proto-\textit{Ethics} has been lost.\textsuperscript{2} However, we know from Oldenburg’s next letter that Spinoza’s geometrical efforts were not entirely successful:

I have received your very learned letter, and read it through with great pleasure. I approve very much of your geometric style of proof, but at the same time I blame my own obtuseness that I do not follow so easily the things you teach so exactly. Please, then, let me give you evidence of my slowness by putting the following Problems to you... (Ep. 3)

Oldenburg’s frustration should be familiar to many readers of the \textit{Ethics} itself. Despite its rigorous Euclidean scaffolding, the \textit{Ethics} remains a largely impenetrable forest to non-specialists. A few pages into the book, a novice might read that a proposition is immediately evident \textit{[patet]} from preceding definitions without having the faintest idea what is even being proven, much less how it follows from the definitions referenced.\textsuperscript{3} Because the text is cumulative, confusions can easily amplify until one is thoroughly lost in the trees. Furthermore, the \textit{Ethics} is the work of a human being of flesh and blood and finite intellect, and as a result it contains quite a few argumentative gaps and a fair amount of ambiguity. The omissions and equivocations do not help comprehension, and one can be left wishing that the text were even more rigorous and Spinoza had elaborated on his positions at key points.

In this paper, our goal is to open a new pathway into the \textit{Ethics} by formally reconstructing an initial fragment of the text using machinery from modern logic. We focus on the first fifteen propositions of Part One—E1p1 to E1p15—together with the definitions and axioms used in their demonstrations. In his opening definitions and axioms, Spinoza...

\textsuperscript{2}In a footnote to his translation Curley reconstructs at least part of the lost geometric enclosure based on discussion in the subsequent letter Ep. 4, and he rightly observes that it resembles more closely the first appendix of the \textit{Short Treatise} (KV) than the completed \textit{Ethics}. Melamed (2019) argues that the first appendix to KV is most probably the earliest draft we currently have of the \textit{Ethics}.

\textsuperscript{3}The first author speaks from his own personal experience in the early days of this project.
introduces the building blocks of his ontology—substance \([\textit{substantia}]\), attribute \([\textit{attributum}]\), mode \([\textit{modus}]\), and God \([\textit{Deus}]\)—and presents several constraints on the ontological, conceptual, and causal relations that obtain between these protagonists. In the propositions themselves, he establishes various core properties of substances, such as that they are self-caused (E1p7) and infinite in their own kind (E1p8). By the time he reaches E1p15, Spinoza has already established his \textit{substance monism}: God, a substance with an infinity of attributes (E1d6), exists (E1p11), and is unique (E1p14), and all inheres in God (E1p15). Not bad for only the opening few pages of the text. Needless to say, just about every one of Spinoza’s central claims in this initial fragment of the \textit{Ethics} has been subject to intense exegetical debate, with commentators remaining far from agreement about how to understand even the most basic foundational elements of Spinoza’s metaphysics.

While one of the main motivations for our project is to make the \textit{Ethics} more accessible to newcomers with a modest background in modal logic, this paper should also be of interest to Spinoza specialists insofar as it can contribute to interpretive debates by clarifying and unpacking the argumentative structure of the text. The strict requirement of formalized proof provides a useful diagnostic tool for identifying tacit premises, redundancies, and potential errors in Spinoza’s \textit{mos geometricus} (spoiler: \textit{pace} Bennett (1984), pp. 16–20, Bennett & van Inwagen (1984), and others, we find Spinoza to be a skilled logician whose argumentation, for the most part, holds together quite well). Formally reconstructing how definitions, axioms, and previously proven propositions come together in the demonstrations of the \textit{Ethics} can also sharpen our understanding of key Spinozian concepts and their interdependencies, and draw critical consideration to terminology that has so far received little attention (such as the notion of things “having nothing in common \([\textit{nihil commune cum se invicem habere}]\)” in E1a5, E1p2, and E1p3).

Because Spinoza was unfamiliar with the logical tools we employ in this paper, some readers might reasonably worry that our study is bound to distort rather than illuminate Spinoza’s nuanced philosophical stances in the \textit{Ethics}. However, what follows is an admixture of informal and formal discussion that tries to significantly engage with debates in the secondary literature, and many of the formal devices we introduce were designed precisely to fit with aspects of Spinoza’s metaphysics. Our formalization project is not simply an extended version of the rote translation exercises familiar from introductory logic courses where one tries to force argumentation into the standard mold of first-order logic; rather, our effort is meant to bring out many subtleties in the logical and conceptual structure of Spinoza’s text, and it is significantly informed by wider Spinoza scholarship. As such, we ask that you withhold judgment on the appropriateness of using modern logical techniques to interpret historical texts like the \textit{Ethics} until you see what work these techniques
can do and are better positioned to gauge the payoffs and downsides of this precise austere methodology—a methodology that strives to achieve the high standards of rigor Spinoza himself aspired to.

It is also worth stressing at the outset that while we take a number of interpretive stands throughout this paper—among these, a new reading of the opening part of the *Ethics* (building on and amending Gueroult 1968, pp. 107–176) according to which Spinoza is theorizing about the unique divine substance from different perspectives or viewpoints—our global, overarching goal is to develop a formal architecture under which exegetical debates can be more precise. You might not be partial to our Gueroult-like reading of the *Ethics* (an interpretation that is more easily articulated once our formal machinery is in place), or you might disagree with other aspects of our reading of the text. No matter. With our formalism at your disposal, you can at least sharpen your dissent.

We begin this project in §2, where we develop a Spinozian model theory for the *Ethics* that provides a semantics for the formal language in which our reconstruction is couched. The reconstruction itself occurs in §§3-6, which is the heart of the paper. We treat definitions E1d1-E1d8 in §3, axioms E1a1-E1a7 in §4, and propositions E1p1-E1p15 together with their demonstrations in §5. This is not the first attempt to formalize the *Ethics*—there is an important historical precedent in Boole’s *Laws of Thought* (1854), where Boole applies his early system of algebraic logic to formalize an opening part of the *Ethics*, and there are some more recent attempts by Jarrett (1978) and others. We compare our account to these earlier efforts in §6 before concluding in §7 with general insights from our logical investigation and some suggestions for further research.

## 2 A Spinozian Model Theory

Here, we work within the framework of *quantified modal logic* (see Fitting & Mendelsohn 1998 for a classic overview). In this framework, a model for a first-order language with modal operators includes a nonempty state space whose members represent different possible states of reality—or “possible worlds”. In standard implementations, each possible world is assigned a domain of quantification consisting of all the entities that exist in this world, and an interpretation function provided by the model maps the nonlogical expressions in the language (constant, function, and predicate symbols) to members of and structures over this domain.

Given Spinoza’s metaphysical views, bringing in the machinery of possible worlds—and, indeed, allowing for domains consisting of multiple entities—might seem like overkill, or even plain distortion. As noted in

---

4Our target readers should have a solid understanding of first-order logic and be familiar with basic methods of deductive proof. We also expect readers to have some exposure to modal logic, though not necessarily of the quantified variety.
the introduction, Spinoza argues for a monism according to which there is only a single substance, God, of which everything else is a mode. Later in Part One, we also take him to establish a necessitarianism according to which every actual state of affairs is necessary—things could not have been otherwise. However, it is important to keep in mind that Spinoza has to argue for these doctrines, and some of his main conclusions are drawn only after a lot of careful preliminary work (see E1p33 and its scholia). So, we don’t want to build too much of Spinoza’s metaphysics directly into our models, which must be capable of representing not only the positions that Spinoza eventually arrives at in the Ethics but also alternative metaphysical possibilities that he rules out through his argumentation, such as universes with multiple co-existing substances and non-necessary facts.\(^5\)

That said, to capture some of Spinoza’s own idiosyncratic views about the universe, our “Spinozian models” have a few non-standard twists. We motivate these in the next two subsections §2.1 and §2.2—and provide more overview of Spinoza’s philosophy in the process—before presenting our full Spinozian model theory in §2.3.

2.1 Modeling the Attributes

At the heart of Spinoza’s ontology is the distinction between substance and mode. The essential hallmark of substance, according to Spinoza, is its independence. In his definition of substance (E1d3), he tells us that substance is “in itself \[in se\]” (i.e., inheres in itself) and “conceived through itself \[per se concipitur\]”. Later, in E1p7d, Spinoza proves that substance is independent in a third sense: substance is the cause of itself and is not caused by any other thing.

In contrast, modes are by their nature dependent beings. In his definition of mode (E1d5), Spinoza asserts that a mode is an “affection” (roughly, a quality) of substance, and then he spells out how modes are dependent in two senses in which substance is not: a mode is “in another \[in alio\] through which it is also conceived \[per alio concipitur\]”—that is, a mode inheres in and must be understood through something other than itself. In E1p16c1, Spinoza also establishes that modes are causally dependent in that they must be caused by another, namely, God.

We will have more to say about the ontological, conceptual, and causal independence/dependence of substance/modes in what follows. For the moment, though, let us turn to the other two protagonists of Spinoza’s metaphysical system—attributes and God—as these raise some pressing interpretative puzzles.

In E1d4, Spinoza defines an attribute as “what the intellect perceives

\(^5\)As we discuss in §2.2, there are also benefits to having metaphysically impossible worlds at our disposal.
of substance, as constituting its essence [id, quod intellectus de substantia percipit, tanquam ejusdem essentiam constituens]”. Spinoza then defines God in E1d6 as “a being absolutely infinite [ens absoluté infinitum]”, which is spelled out further as “a substance consisting of an infinity of attributes, of which each one expresses an eternal and infinite essence”. Here Spinoza builds on but departs markedly from Descartes, who writes in his Principles of Philosophy:

To each substance there belongs one principal attribute; in the case of mind, this is thought, and in the case of body it is extension. A substance may indeed be known through any attribute at all; but each substance has one principal property which constitutes its nature and essence, and to which all its other properties are referred. Thus extension in length, breadth, and depth constitutes the nature of corporeal substance; and thought constitutes the nature of thinking substance. (I. 53)

Unlike Descartes, who rules out the possibility of one substance having more than one principal attribute (and who allows for a multiplicity of co-existing substances), Spinoza allows substances—or rather the one divine substance, God—to have multiple attributes. Of God’s many attributes, we humans have access to only two: Thought and Extension (see E2a5, E2p13, and Ep. 64). But God has infinitely more attributes beyond our epistemic and causal reach (E1d6 and Ep. 56 (IV/261/14)).

Crucially, attributes furnish Spinoza’s metaphysics with diversity while allowing Spinoza to maintain his substance monism. However, it is not immediately clear how Spinoza pulls off this balancing act, as his definitions of attribute and God trigger a cascade of questions. Why does Spinoza refer to the intellect in his definition of attribute (E1d4)? What role is it playing exactly? Is he referring to a finite or to an infinite intellect? And what exactly is being perceived by this intellect? What constitutes the essence of substance? Turning to the definition of God (E1d6), in what sense does the divine substance consist of an infinity of attributes? Are attributes parts of God? Are attributes substances?

Before beginning to develop our formal apparatus, we want to simply lay out our preferred interpretation of Spinoza’s theory of attributes (we have more to say about E1d4 and E1d6 in the next section). Though we have neither time nor space to motivate this interpretation over its many competitors, it is useful to have a firm proposal in mind as we proceed.

---

6For further discussion of Descartes’ theory of attributes and the extent to which Spinoza was influenced by it, see Melamed (2018a).

7Bennett (1984), pp. 75–79, infamously claims that by “an infinity of attributes” in E1d6, Spinoza means merely all possible attributes, and that this is compatible with there being only the two attributes of Thought and Extension. However, see Melamed (2018b) for a detailed critique of Bennett’s reading.
Moreover, much of the formal structure in our model theory can in any case be adapted to other possible answers to the above questions—and so is compatible with a range of different interpretations of the Spinozian God and its attributes.

A common, though oversimplified, taxonomy divides interpretations of Spinoza’s attributes into two camps: subjectivist and objectivist. While the subjective position goes back to Hegel, the locus classicus is Wolfson (1934) where Spinoza is taken to claim that attributes are mere inventions of the finite perceiving mind:

If the expression ‘which the intellect perceives’ is laid stress upon, it would seem that the attributes are only in intellectu. Attributes would thus be only a subjective mode of thinking, expressing a relation to a perceiving subject and having no real existence in the essence [of substance]. According to [this] interpretation, to be perceived by the mind means to be invented by the mind. (vol. 1, p. 146)

Because Wolfson’s reading has been subjected to devastating (and to our mind justified) critique, we mention it only to set it aside. This isn’t to say that Wolfson was misguided in calling readers’ attention to the role of the intellect in E1d4. However, we disagree with Wolfson’s view that this role is one of invention and, in effect, that the multiplicity afforded by God’s attributes isn’t grounded in reality but rather is confined to perceptions of a finite intellect.

Our own working position is objectivist, in at least the sense that we do not take attributes to be inventions of the human mind. This position is largely based on Garrett (2017), because we think this is the best interpretation currently on the market. Following Garrett (and echoing Melamed 2012a), we regard Spinoza as a proponent of a “strong ontological pluralism” according to which one thing can have more than one “fundamental manner or kind of existence, reality, or being” (what

---

\(^8\)Our own interpretation sketched below, which we classify as “objectivist”, might also be taken to have a “subjectivist” component in that we still take the intellect to play a non-trivial role.

\(^9\)See Gueroult (1968), pp. 441–61, and Haserot (1972) for objections to Wolfson. One of the most damning objections is that the illusory nature of the attributes in the subjectivist interpretation is at odds with Spinoza’s doctrine that the intellect, either finite or infinite, perceives things adequately and it is only the imagination that can be the source of error (E2p41). Thus, the intellect’s perception of attributes cannot be an error that fails to reflect the true nature of substance. To make matters worse, Spinoza rephrases his definition of attribute in E2p7s, referring to an attribute as “whatever can be perceived by an infinite intellect as constituting the essence of substance” (italics added). So, apparently, Spinoza has God’s infinite intellect in mind in E1d4 and the subjectivist is committed to saying that God’s knowledge of itself is an illusion.
Melamed calls “aspects” of God’s existence). The idea that there can be more than one kind of existence—this is the “ontological pluralism” part—might not strike at least some philosophers as especially peculiar given the distinction between concrete and abstract objects, particulars and universals, or the divine and mundane. On Garrett’s interpretation, however, Spinoza takes this further in proposing that a single thing can have existence of several different kinds—this is the “strong” part. Indeed, God is a substance having infinitely many fundamental kinds of existence, each of which might be regarded as one of God’s attributes. Spinoza’s “thinking substance” (mentioned in E2p7s) is God existing as a thinking thing, “extended substance” (mentioned in E1p15s and E2p7s) is God existing as extended, and likewise for the other unknown attributes shrouded in darkness—they too might be regarded as different kinds of existence (from E1p20d: “each of [God’s] attributes expresses existence”). The same goes for finite things: on Spinoza’s ingenious solution to the Mind-Body Problem, your mind is you existing as a thinking thing while your body is you existing as extended (E2p21s, E3p2s).

While Garrett doesn’t emphasize the role of the intellect in all this, we take E1d4 to impose a substantive condition on fundamental kinds of existence—these kinds correspond to how substance and its modes can be perceived by the infinite intellect as fundamental (see note 9). Adopting one perspective, the intellect perceives God as a thinking thing (E2p1) and its various modes as modes of thought. Adopting another perspective, the intellect perceives God as an extended thing (E2p2) and its modes as modes of extension. Yet, contra Wolfson, the kinds of existence shared by God and its modes aren’t invented or produced.

---

10 Because Garrett’s talk of “manners of existence” is liable to lead at least some readers to mistakenly identify these manners with Spinozian modes, we avoid this terminology and talk only of “kinds of existence”.

11 Garrett suggests that Spinoza is appropriating and generalizing the Scholastic distinction between formal and objective being, conceived as two genuine kinds of existence or reality.

12 That being said, as Jarrett (1978) observes, the role of the intellect in E1d4 falls out of view in E1p20d.

13 In his early correspondence (Ep. 2, IV/7/27-29, and Ep. 4, IV/14/9-10), Spinoza cites his definitions of attribute (and substance) from early drafts of the Ethics. Notably, these early definitions of attribute make no mention or reference to the intellect. Then, in Ep. 9 (dated March 1663), Spinoza quotes the following definition from another draft that this time stresses the attributes’ relation to the intellect:

By substance I understand what is in itself and is conceived through itself, i.e., whose concept does not involve the concept of another thing. I understand the same by attribute [idem per attributum intelligo], except that it is called attribute in relation to the intellect [respectu intellectus], which attributes such and such a definite nature [certam talam naturam] to substance. (IV/46/20, italics added)

These “definite natures” are, on the current proposal, fundamental kinds of existence.
by the intellect; God’s multifaceted crystalline reality is there all along (from E1p10s: “All the attributes [of substance] have always been in it together”).

To formally capture Spinoza’s idea that one and the same thing can have existence of many kinds—though, to repeat, we think our formalism is to an extent detachable from the above informal gloss in that parts of it might be used to model other interpretive avenues regarding attributes—we adopt a many-sorted model theory (Schmidt 1938, 1951; Wang 1952; see Manzano 1996 or Väänänen 2014 for a recent overview). Unlike single-sorted models for modal logic, which include a set of possible worlds \( \mathcal{W} \) where each world \( w \in \mathcal{W} \) is assigned a single domain \( D(w) \) consisting of the entities that exist in this world, many-sorted models assign a potentially infinite number of domains of quantification to each world. Where \( S = \{s_1, s_2, \ldots\} \) is an index set of sorts, a many-sorted model \( \mathcal{M} \) for \( S \) assigns each world \( w \in \mathcal{W} \) a domain \( D_{s_1}(w) \) of existents of sort \( s_1 \), a domain \( D_{s_2}(w) \) of existents of sort \( s_2 \), and so forth.\(^\text{15}\) Given a sort \( s \in S \), the variables \( x_s, y_s, \ldots \) range over things of this sort and \( \forall x_s, \exists y_s, \ldots \) quantify over \( D_s(w) \). While entities in different domains in a many-sorted model are generally regarded as distinct, we repurpose these models to allow for one and the same thing to exist in multiple sortal domains.

For purposes of modeling Spinoza’s metaphysics, we work with an infinite set of sorts that includes the sort \( \text{Th} \) of thinking entities (or rather, entities existing as thinking), the sort \( \text{Ex} \) of extended entities (or entities existing as extended), and sorts corresponding to all the other attributes. We call these sorts tied to specific attributes “secondary sorts” because we also postulate a “primary sort” \( \aleph \) whose domain \( D_\aleph(w) \) at a world \( w \in \mathcal{W} \) consists of all the entities having any kind of existence in \( w \) conceived in their fullness as multifaceted beings:

\[
\text{Spinozian sorts} \quad \mathcal{S}_{\text{Spinoza}} = \{\aleph, \text{Th}, \text{Ex}, \ldots\}
\]

The primary \( \aleph \)-sort affords a bird’s-eye view of a pluralistic ontology. To theorize at this global layer is not to substract (or abstract away) the attributes from substance and its modes because on the interpretation we work with, to strip away all the attributes of a thing would be to

\(^{14}\) Apparently, for this reason Spinoza defines an attribute in E1d4 as what the intellect perceives rather than conceives of substance as constituting its essence. As Spinoza discusses in E2d3e, conception involves mental activity whereas perception involves mere responsiveness of the mind to what is already in res. The extent to which Spinoza adheres to the distinction between conception and perception in the later parts of the Ethics is a subject requiring further study.

\(^{15}\) We work with variable domain models in which entities that exist in some worlds can fail to exist in others. Among other things, this feature of our models will enable us to capture Spinoza’s view that the essences of modes do not settle whether they exist, and so it is conceivable both that they exist and that they do not.
deny it existence of any kind. On the contrary, entities are regarded from the “ℵ-perspective” as having all their attributes—for instance, Spinoza adopts this all-encompassing perspective in defining God as a being consisting of an infinity of attributes in E1d6.

To help keep track of the identity of entities across the different sortal domains in a Spinozian model for $S_{\text{Spinoza}}$, we assume that such a model comes equipped with a family of (partial) projection functions $\pi_{\text{Th}}$, $\pi_{\text{Ex}}$, ... that at each world $w \in W$ “project” the multifaceted beings dwelling in the primary domain $D_{\text{ℵ}}(w)$ of this world to single-faceted entities in its secondary sortal domains (to project a multifaceted thing is to home in on its having existence of this or that kind):

**Projection into secondary domains**

For any world $w \in W$ and secondary sort $s \in \{\text{Th}, \text{Ex}, \ldots\}$, the projection function $\pi_s(w)$ maps entities from $D_{\text{ℵ}}(w)$ into $D_s(w)$. When defined, $\pi_s(w)(a_\text{ℵ})$ is $a_\text{ℵ}$ in $w$ as perceived by the intellect as a being of sort $s$.

For instance, if ‘$\text{God}_{\text{ℵ}}$’ denotes Spinoza’s absolutely infinite substance at $w$, then $\pi_{\text{Th}}(w)(\text{God}_{\text{ℵ}})$ is the thinking substance (i.e., $\text{God}_{\text{Th}}$), $\pi_{\text{Ex}}(w)(\text{God}_{\text{ℵ}})$ is the extended substance (i.e., $\text{God}_{\text{Ex}}$), and so forth. Because any single entity is singular under any kind of existence, and the primary domain $D_{\text{ℵ}}(w)$ of a world $w$ includes all the entities with any kind of existence in $w$, we require that each $\pi_s(w)$ is a one-one injective function, each member of a secondary domain $D_s(w)$ is the $\pi_s(w)$-projection of some member of $D_{\text{ℵ}}(w)$, and (in the other direction) every multifaceted being $a_\text{ℵ} \in D_{\text{ℵ}}(w)$ is projected into at least one of the secondary domains of $w$—that is, $\pi_s(w)(a_\text{ℵ})$ is defined for some $s \in \{\text{Th}, \text{Ex}, \ldots\}$.

Given the different sortal layers in Spinozian models, we can identify a number of (genuine) identity relations. First, there is the “standard” identity relation $=$ that any element in any domain of a model stands in with respect to itself and to no other element (i.e., $=$ is the diagonal relation over these quantificational domains):

**Standard identity**

$a_s = b_s$ at $w$ iff $a_s, b_s$ are the same element in a model.

Second, there is what we call the “projective” identity of any multifaceted being in the ℵ-domain with each of its projections. For $s \in \{\text{Th}, \text{Ex}, \ldots\}$,

**Projective identity**

$a_\text{ℵ} =_P b_s$ at $w$ iff $\pi_s(w)(a_\text{ℵ}) = b_s$.\(^\text{17}\)

\(^{16}\)In his *Study of Spinoza’s Ethics*, Bennett postulates the existence of a layer of trans-attribute qualities that do not belong to any attribute (pp. 145–7). For a clear refutation of this reading, see Wilson (1981), pp. 585–6.
Third, there is the “cross-attribute” or “trans-attribute” identity of these projections. For \( s, s' \in \{ \text{Th, Ex, ...} \}, \)

**Cross-attribute identity**

\[
a_s =_C b_{s'} \text{ at } w \iff \text{there is some } c_\aleph \text{ such that } c_\aleph =_P a_s \text{ and } c_\aleph =_P b_{s'} \text{ at } w. \tag{18} \]

All three relations are genuine identity relations in the sense that if \( a_s = b_{s'}, a_s =_C b_{s'}, \) or \( a_s =_P b_{s'}, \) then, in an important sense, \( a_s \) and \( b_{s'} \) are *one and the same thing*, though \( a_s \) and \( b_{s'} \) might still differ in terms of their kind or kinds of existence. In monistic ontologies involving only a single kind of existence, there is room for only a single notion of identity. However, in a pluralistic setting where one is referring to and quantifying over entities with more than one fundamental kind of existence, it is useful to have multiple notions of identity in play to capture more fine-grained notions of sameness and difference. \(^{19}\)

Note that the three identity relations differ with respect to their logical properties. Standard identity \( = \) is an equivalence relation—it is reflexive, symmetric, and transitive. \(^{20}\) So, too, is cross-attribute identity on the secondary domains where it applies. However, projective identity is neither reflexive nor symmetric, and is transitive only in a vacuous sense as we can never have \( a_s =_P b_{s'} \) and \( b_{s'} =_P c_{s''} \) for any \( a_s, b_{s'}, c_{s''}. \)

That said, we can get failures of transitivity if we consider *combinations* of our identity relations. As discussed above, we can have \( \text{God}_{\aleph} =_P \text{God}_{\text{Th}} \) (E2p1: “God is a thinking thing”) and \( \text{God}_{\aleph} =_P \text{God}_{\text{Ex}} \) (E2p2: “God is an extended thing”), but while \( \text{God}_{\text{Th}} =_C \text{God}_{\text{Ex}}, \text{God}_{\text{Th}} \neq \text{God}_{\text{Ex}}. \)

This transitivity failure reveals how standard identity is a stricter notion than cross-attribute identity over the secondary domains where these notions both apply. Cross-attribute identity is the appropriate notion of identity when we are counting substances and modes in the ontology but are not concerned with the distinction between different kinds of existence. From this coarse-grained perspective, \( \text{God}_{\text{Th}} \) and \( \text{God}_{\text{Es}} \) are numerically identical because we are talking about one and the same substance—and since the locutions “thinking substance” and “extended substance” lend themselves to counting substances in the ontology (as Garrett 2017 observes), Spinoza is happy to say in E2p7s that “the

---

\(^{17}\)This equivalence needn’t be regarded as a *definition* of projective identity in terms of standard identity. We do not mean to claim that standard identity is more fundamental than projective identity.

\(^{18}\)Again, this equivalence needn’t be regarded as a definition. We do not mean to claim that projective identity is more fundamental than cross-attribute identity.

\(^{19}\)Within our formal framework, one could think of the \( \aleph \)-domain of a world as the *quotient set* determined by \( =_C \) on the union of the secondary sortal domains. Elements in the \( \aleph \)-domain would then be equivalence classes of cross-attributively identical elements in the secondary domains.

\(^{20}\)A relation \( R \) is reflexive iff \( Ra_a \) for all \( a \), symmetric iff \( Ra_b \) implies \( Rb_a \) for all \( a, b \), and transitive iff \( Ra_b \) and \( Rb_c \) implies \( Rb_c \) for all \( a, b, c \).
thinking substance and the extended substance are one and the same substance \[ \text{una eademque est substantia} \] . On the other hand, standard identity is the appropriate notion when we are counting things as distinct when they have different kinds of existence. From this more fine-grained perspective, \( \text{God}_\text{Th} \) and \( \text{God}_\text{Ex} \) are numerically distinct because the former is God existing as thinking while the latter is God existing as extended.

Our identity relations also differ with respect to their substitutional properties. All the predicates introduced in this paper are referentially transparent contexts with respect to standard identity. For instance, where ‘Extended(\( t \))’ and ‘Affection(\( t, t' \))’ formalize \( t \) is extended and \( t \) is an affection of \( t' \) respectively, the following conditionals hold:

\[
\begin{align*}
\text{If } \text{Extended}(a_s) \text{ and } a_s &= b_s', \text{ then } \text{Extended}(b_s'). \\
\text{If } \text{Affection}(a_s, b_s'), a_s &= c_s', \text{ and } b_s' &= d_s'', \text{ then } \text{Affection}(c_s', d_s'').
\end{align*}
\]

In models that accurately capture Spinoza’s philosophy, many predicates will also turn out to be transparent with respect to projective identity in the restricted sense that if they hold with respect to some elements in the primary domain \( D_\text{R}(w) \), and these elements are all projected into the same secondary sortal domain \( D_s(w) \), then the predicates hold with respect to these projections as well. For instance, where ‘Substance(\( t \))’ and ‘\( t \rightsquigarrow t' \)’ formalize \( t \) is a substance and \( t \) causes \( t' \) respectively, the following conditionals hold:

\[
\begin{align*}
\text{If } \text{Substance}(a_\text{R}) \text{ and } a_\text{R} &= b_\text{s}, \text{ then } \text{Substance}(b_\text{s}). \\
\text{If } a_\text{R} \rightsquigarrow b_\text{R}, a_\text{R} &= b_\text{R} =_p c_\text{s}, \text{ and } b_\text{R} =_p d_\text{s}, \text{ then } c_\text{s} \rightsquigarrow d_\text{s}.
\end{align*}
\]

On the other hand, many predicates will be referentially opaque contexts for projective identity in its full generality—for example, we can have \( \text{Affection}(a_\text{R}, b_\text{R}), a_\text{R} =_p c_\text{Ex}, b_\text{R} =_p d_\text{Th}, \text{ but } \lnot \text{Affection}(c_\text{Ex}, d_\text{Th}) \). As Della Rocca (1993, 1996) observes, “attribute contexts” like ‘Extended(\( t \))’ and related attribute-sensitive predicates are also referentially opaque with respect to cross-attribute identity (though Della Rocca does not phrase his observation in these terms)—for example, we can have \( \text{Extended}(a_\text{Ex}), a_\text{Ex} =_C b_\text{Th}, \text{ but } \lnot \text{Extended}(b_\text{Th}) \).

### 2.2 Modeling Possibility and Conceivability

While Spinoza’s modal metaphysics remains the subject of considerable debate (see Newlands 2018a,b for helpful discussion and references), we interpret him as a strict necessitarian. This commitment is strongly suggested in various places in the Ethics, such as in E1p29, where Spinoza proves that “in nature there is nothing contingent”. Four propositions

\[ \text{We interpret Spinoza literally as using ‘una eademque est substantia’ to express the numerical identity of substance.} \]

\[ \text{See Garrett (2017) for closely related discussion of the status of the transitivity of identity within Spinoza’s philosophy.} \]
later, he also proves the following:

P33: Things could have been produced by God in no other way, and in no other order than they have been produced.

Dem: For all things have necessarily followed from God’s given nature (by P16), and have been determined from the necessity of God’s nature to exist and produce an effect in a certain way (by P29). Therefore, if things could have been of another nature, or could have been determined to produce an effect in another way, so that the order of Nature was different, then God’s nature could also have been other than it is now, and therefore (by P11) that [other nature] would also have had to exist, and consequently, there could have been two or more Gods, which is absurd (by P14C1). So things could have been produced in no other way and no other order, etc., q.e.d.

Having already established that the existence and activity of all things is “determined from the necessity of the divine nature” (E1p29; see also E1p16, Ep. 43, IV/221/34 and Ep. 75, IV/311/19), Spinoza runs the following reductio argument in the above demonstration: if the order of Nature [ordo naturae] could have been different, then so too could have been God’s essence or nature, and because any alternative God would have to exist (E1p11), we would end up with multiple Gods, contradicting his substance monism (E1p14).24

Even though Spinoza is a necessitarian, there are still good reasons to have multiple worlds available in our models. First, as mentioned, Spinoza argues for his necessitarianism only in the second half of Part One—the argumentation only really gets going in E1p16, where this paper leaves off—so we don’t want to presuppose this doctrine in our model theory, which should be capable of representing rival views. While any model that accurately incorporates Spinoza’s modal commitments will be one in which the actual world is the sole metaphysical possibility, we allow for models that include more than one metaphysically possible world in order to represent alternative views that Spinoza rejects.

Furthermore, even in models encoding Spinoza’s necessitarianism wherein actuality and metaphysical possibility coincide, there are also benefits to having metaphysically impossible worlds lying around. With such worlds, we can capture Spinoza’s rich modal metaphysics and more

---

23For Spinoza, ‘nature’ and ‘essence’ are frequently interchangeable.

24Curley argues that Spinoza is merely a determinist, not a strict necessitarian—that is, Spinoza is not committed to the rejection of alternative possible worlds so long as these alternative worlds are ordered in a deterministic manner (Curley 1969, pp. 82–117, and Curley & Wlaski 2015). For a conclusive refutation of this claim, see Garrett (1991, 2017). See also Newlands (2018a,b) for helpful discussion.
nuanced necessitarianism, which asserts not simply that things could not have been otherwise but that things could not have been otherwise by virtue of God’s essence—the full natural order flows from the necessity of God’s essence (E1p16). In the first scholium immediately following E1p33d, Spinoza goes on to distinguish between two different sources or grounds of the necessary existence and nonexistence of things:

A thing is called necessary either by reason of its essence or by reason of its cause. For a thing’s existence follows necessarily either from its essence and definition or from a given efficient cause. And a thing is also called impossible from these same causes—namely, either because its essence, or definition, involves a contradiction, or because there is no external cause which has been determined to produce such a thing.

For Spinoza, everything that exists necessarily exists and everything that does not exist necessarily fails to exist, but there are different reasons that things are necessarily ruled into or out of existence. The existence and nonexistence of some things is necessitated by their own essence or nature. In E1p7, for instance, Spinoza proves that “it pertains to the nature of a substance to exist”. As for nonexistence, there are square circles and other “Chimeras” that fail to exist by virtue of their essence (E1p11d2). In contrast, the existence or nonexistence of other things is due to their (external) efficient cause—thus, for example, the existence of a broken window (and the nonexistence of a non-broken window) might be due to the impact of a rock crashing through it. More generally, we can think of the essence or real definition of a thing (or things) as settling certain subject matters while leaving open how things stand with respect to other matters. Spinoza argues that God’s essence necessitates the full ordo naturae, but the essence of any nonsubstance, taken by itself, must leave many subject matters unsettled, such as the matter of this nonsubstance’s own existence (E1p24).

Though we might need as few as one possible world to represent the metaphysically possible, the abundance of worlds in our Spinozian models is helpful for modeling what is necessitated by the essences of things and what is possible relative to these essences, which for Spinoza can outstrip the metaphysically possible. At this point, let us assume that the domain assignments $D_N$, $D_{Th}$, $D_{Ex}$, ... in a Spinozian model $M$ map each world $w \in W$ to sets of existents (“local” domains) drawn from “global” domains $D_N$, $D_{Th}$, $D_{Ex}$, ... of the respective sorts; that is, $D_s(w) \subseteq D_s$ for each $s \in S_{Spinoza}$. The global domains in a model include

\footnote{Spinoza famously endorses a strong version of the Principle of Sufficient Reason (PSR) according to which there is a cause or reason for the existence or nonexistence of each thing (E1p11d2; more on the PSR in §5).}
all the things we wish to theorize about, whether existent or nonexistent, possible or impossible, conceivable or inconceivable—a global domain $\mathcal{D}$, can even include Chimeras like square circles, mountains without valleys, and so forth. Let $\mathcal{D}^*$ be the union of the global domains. We assume that along with the world-internal structure already introduced in §2.1 to implement Spinoza’s theory of attributes, a Spinozian model includes an essence function $\mathcal{E}$ that assigns to each element $a \in \mathcal{D}^*$ the set of propositions (i.e., a set of sets of worlds; Hintikka 1962; Stalnaker 1984) necessitated or forced by its essence:

**Necessities by reason of essence**

The essence function $\mathcal{E}$ maps each thing $a \in \mathcal{D}^*$ to a set of propositions $\mathcal{E}(a) \subseteq \mathcal{P}(\mathcal{W})$, where $P \in \mathcal{E}(a)$ iff $P$ is true in virtue of $a$’s essence.\(^{26}\)

The worlds in the intersection $\cap \mathcal{E}(a)$, which we call the “essence set” of $a$, are compatible with every proposition necessitated by $a$’s essence or nature whereas worlds outside this intersection are excluded by $a$’s essence. Spinoza’s claim in E1p7 that it pertains to the essence of a substance to exist entails that if $a$ is a substance, then $\mathcal{E}(a)$ includes the proposition that $a$ exists, and therefore every world in $\cap \mathcal{E}(a)$ is one in which $a$ exists. In contrast, if $a$ is a nonsubstance (i.e., $a$ is a mode (E1p4d)), and thus, for Spinoza it exists by virtue of its efficient causes (E1p24), then $\mathcal{E}(a)$ cannot include the proposition that $a$ exists, and therefore $\cap \mathcal{E}(a)$ can include worlds in which $a$ fails to exist.

As for metaphysical possibility itself, we further assume that what is metaphysically possible is dependent on what is possible with respect to essences—in particular, to be metaphysically possible is to be possible relative to the essences of all things (in fact, for Spinoza only God’s essence need be taken into account):

**Metaphysical possibility**

$w$ is a metaphysical possibility $\equiv w \in \cap \mathcal{E}(a)$ for each $a \in \mathcal{D}^*$.

Introducing the name ‘@’ for the actual world in a model, we require that @ $\in \mathcal{W}$ be possible relative to the essence of any thing, which ensures that the actual world is metaphysically possible:

**Actual is possible:** @ $\in \cap \mathcal{E}(a)$ for each $a \in \mathcal{D}^*$.

Spinoza’s claim that God’s essence fixes the full order of Nature can be captured by the requirement that $\cap \mathcal{E}(\mathsf{God}) = \{ @ \}$. This enforces that @ is the only metaphysically possible world in the model. But, again, Spinoza has to argue for this position, and so we also allow for models in

---

\(^{26}\)To represent what follows from the essences of multiple things taken together, one could define essence functions on the set of nonempty subsets of $\mathcal{D}^*$ rather than on $\mathcal{D}^*$ itself. However, this extra structure is unnecessary for what follows.
which God’s essence together with the essences of all other things leaves open multiple metaphysical possibilities.

To summarize, there are three kinds of worlds in Spinozian models. First, there are metaphysically possible worlds, such as the actual world @, which are compatible with the essences of all things. Second, there are metaphysically impossible worlds which are compatible with the essence of no thing. These worlds will not play an important role in what follows and can be disregarded. Third, there are metaphysically impossible worlds which, though ruled out by the essences of all things when taken together, are nevertheless compatible with the essence of some particular thing (or things) and can therefore be used to capture how this essence leaves various subject matters unsettled. A metaphysically impossible world \( w \in W \) lying in the essence set \( \bigcap E(a) \) of some \( a \in \mathcal{D}^* \) might still be regarded as an open possibility in the restricted sense that the essence of a alone (which needn’t include its full causal history) doesn’t rule out this world.

This brings us to the notion of conceivability, which is intimately related to essence-relative modality in Spinoza’s philosophy and appears in several key texts at the beginning of the *Ethics* (see for example E1d1, E1a7, E1p10s, E1p11d1, and E1p14). At least in his early period, Spinoza seems to think that conceivability amounts to the possibility of positing certain ideas in an infinite intellect. In the first appendix to *Korte Verhandeling*, he proves the following proposition:

\[
P4: \text{Existence belongs, by nature, to the essence of every substance, so much so that it is impossible to posit in an infinite intellect the idea of the essence of a substance which does not exist in nature. (italics added)}
\]

Compare this with E1d1 in the *Ethics*, where a “cause of itself [causa sui]” (a substance, as proven in E1p7) is variously defined as that “whose essence involves existence” and that “whose nature cannot be conceived except as existing” (our emphasis). Spinoza also posits in E1a7 that

---

27 These metaphysically impossible worlds might still be regarded as epistemically possible. After the passage from E1p33s1 quoted above, Spinoza continues as follows:

But a thing is called contingent only because of a defect of our knowledge. For if we do not know that the thing’s essence involves a contradiction, or if we do know very well that its essence does not involve a contradiction, and nevertheless can affirm nothing certainly about its existence, because the order of causes is hidden from us, it can never seem to us either necessary or impossible. So we call it contingent or possible.

While this notion of epistemic possibility will not play an important role in what follows—unlike the notion of conceivability, which we discuss shortly—it is crucial to Spinoza’s theory of human psychology and the supervening disciplines of ethics and political philosophy developed in the later parts of the *Ethics*. 
anything conceivable as not existing has an essence that does not involve existence. So, more generally, he seems to think that if the essence of a thing necessitates certain facts about this thing—such as the fact that it exists—then the thing cannot be conceived in ways that conflict with these essentialist facts—it cannot be conceived as not existing. For a mundane example, assuming that it is an essentialist fact about water that it is H$_2$O, Spinoza would take it to be inconceivable that water has a different chemical composition.

Importantly, Spinoza is concerned in E1d1 and E1a7 (and elsewhere in the fragment of the Ethics under consideration) with what can be conceived about a thing when attending to only its essence. Newlands (2018a), pp. 25–28, helpfully distinguishes between the “narrow concept” (NC) of a thing that includes only its essential “intrinsic” features and its “broad concept” (BC) that also includes its relation to each of its causes. The kind of conceivability at issue here is that involving NCs. Though one might ultimately want to do something fancy and model this narrow conceivability in terms of the possibility of positing ideas in an infinite intellect, we want to remain fairly noncommittal about how Spinoza understands conceivability in his later philosophy, and we also want to keep the formalism relatively simple. So, we hardwire conceivability into our Spinozian models by taking them to include a conceivability function $C$ that assigns to each member $a \in \mathcal{D}^*$ the set of propositions conceivable about it in the narrow sense, which we call the “conceivability set” of $a$:

**Conceivability**

The conceivability function $C$ maps each thing $a \in \mathcal{D}^*$ to a set of propositions $C(a) \subseteq \mathcal{P}(\mathcal{W})$, where $P \in C(a)$ iff $P$ is conceivable about $a$ when considering only $a$’s essence or NC.

Spinoza’s notion of conceivability is a rich topic that requires a great deal more attention than we can offer here (see Newlands 2018a for further discussion). In this paper, we concern ourselves mainly with the links that Spinoza forges in the beginning of the Ethics between conceivability and related notions. Particularly important is the connection between what pertains to the essence of a thing and what is conceivable about it given its NC, which can be made formally precise using the essence and conceivability functions in our models. For instance, E1a7 requires that if the proposition that $a$ does not exist lies in the conceivability set $C(a)$ (“If a thing can be conceived as not existing...”), then the proposition that $a$ exists is not a member of $\mathcal{E}(a)$ (“...its essence does not involve existence”), and the essence set $\bigcap \mathcal{E}(a)$ can include worlds in which $a$ fails to exist. More generally, Spinoza’s idea that what is narrowly conceivable about a thing must be compatible with its essence might be spelled out formally.

---

28To represent what is conceivable about collections of things given their NCs, one could define conceivability functions on the set of nonempty subsets of $\mathcal{D}^*$. However, this extra structure is unnecessary.
in terms of the set-theoretic requirement that every proposition in its conceivability set overlaps with its essence set.

2.3 The Full Spinozian Model Theory

In this subsection, we present our complete Spinozian model theory, the semantic backbone of our project. While this model theory is of necessity formally involved, and can be skimmed by readers eager to get started marching through the Ethics, readers should at least familiarize themselves with the formal Spinozian language that we lay out below and the main contours of its model theory. The fine-grained semantic details can be left for future reference in the event that any of the formalism in subsequent sections is unclear.

To represent the logical forms of sentences in the initial fragment of Ethics up through E1p15, we adopt a language whose logical symbols include the standard sentential connectives (negation ‘¬’, conjunction ‘∧’, disjunction ‘∨’, the material conditional ‘→’ and biconditional ‘≡’), the actualist quantifiers ‘∀’ and ‘∃’, the possibilist quantifiers ‘Π’ and ‘Σ’, and variables indexed to every Spinozian sort. For quantificational purposes, we also include unindexed variables ‘x’, ‘y’, ... and overlined unindexed variables ‘¯x’, ‘¯y’, ... in this object language for denoting things of any Spinozian sort in $S_{Spinoza}$ and of any secondary sort in $\{Th, Ex, ...\}$ respectively. Whereas standard first-order languages have only a single symbol for identity, we now have three:

**Identity symbols:** ‘=’, ‘=P’, and ‘=C’ for standard, projective, and cross-attribute identity

The language also includes the following sentential modal operators:

**Necessary-by-essence operators:** ‘□t’ (read: *It is necessitated by the essence of t that...*) for each term t (i.e., constant or variable) in the language

**Metaphysical necessity/possibility operators:** ‘□’ (read: *It is metaphysically necessary that...*) and ‘♦’ (read: *It is metaphysically possible that...*)

**Conceivability operators:** ‘♦t’ (read: *It is conceivable about t when conceived solely in terms of its essence that...*) for each term t in the language

---

29 See Fitting & Mendelsohn (1998) for an overview of actualist vs. possibilist quantification.

30 Cf. the relativized modal operators in Fine’s (1995; 2000) *logic of essence*. One could also introduce additional modals to express what is necessary/possible relative to essence sets, but this expressibility is unnecessary for what follows.
Rounding out the logical vocabulary are the following predicate symbols for talking about the sortal-projective structure of a Spinozian model (the sortal subscripts or lack thereof on argument positions indicate whether they can be instantiated by things of any sort \( t \), multifaceted things only \( t_\aleph \), single-faceted things only \( \bar{t} \), or things of some specific secondary sort \( t_{\text{Thr}}, t_{\text{Ex}}, \ldots \)):

- **Same-sort**\((t, t')\): \( t \) and \( t' \) are the same sort of thing
- **All-sorts**\( (t_\aleph) \): \( t_\aleph \) is projected into each of the infinitely many secondary sortal domains

Turning to the nonlogical vocabulary of the language, the remaining symbols can be divided into two categories: undefined and defined. The first category of undefined predicates includes the following three binary relation symbols for Spinoza’s core metaphysical relations:

- \( t \leq t' \): \( t \) inheres in \( t' \)
- \( t \sqsubseteq t' \): \( t \) is conceived through \( t' \)
- \( t \rightsquivalence t' \): \( t \) causes \( t' \)

Let ‘\( t < t' \)’ abbreviate ‘\( t \leq t' \land t' \notin t \)’, ‘\( t \sqsubseteq t' \)’ abbreviate ‘\( t \sqsubseteq t' \land t' \not\sqsubseteq t \)’, and ‘\( t \rightsquivalence t' \)’ abbreviate ‘\( t \rightsquivalence t' \land t' \rightsquivalence t \)’. The Spinozian language also includes the following undefined predicates for other primitive notions occurring in the early *Ethics*:

- \( t \mid t' \): \( t \) is limited by \( t' \)
- \( t \perp t' \): \( t \) and \( t' \) have nothing in common
- **True-idea**\( (t_{\text{Th}}) \): \( t_{\text{Th}} \) is a true idea
- **Agree**\( (t_{\text{Th}}, t) \): \( t_{\text{Th}} \) agrees with \( t \)
- **Object**\( (t, t_{\text{Th}}) \): \( t \) is the object of \( t_{\text{Th}} \)
- **D**\( (t, t', t'', t''') \): \( t \) and \( t' \) are distinguished by the difference between \( t'' \) and \( t''' \)
- \( t_\aleph \leq_R t_\aleph \): \( t_\aleph \) has at least as much reality or being as \( t_\aleph \)
- \( t_\aleph \leq_A t_\aleph \): \( t_\aleph \) has at least as many attributes belonging to it as \( t_\aleph \)
- **Divisible**\( (t) \): \( t \) is divisible
- **Part**\( (t, t') \): \( t \) is part of \( t' \)

In E1d1-E1d8, Spinoza defines the following predicates:

- **Causa-sui**\( (t) \): \( t \) is a cause of itself
- **Finite-in-kind**\( (\bar{t}) \): \( \bar{t} \) is finite in its own kind
- **Substance**\( (t) \): \( t \) is substance
- **Attribute**\( (\bar{t}) \): \( \bar{t} \) is an attribute
- **Mode**\( (t) \): \( t \) is a mode
- **Affection**\( (t, t') \): \( t \) is an affection of \( t' \)
- **God**\( (t_\aleph) \): \( t_\aleph \) is God
- **Abs-infinite**\( (t_\aleph) \): \( t_\aleph \) is absolutely infinite
- **Free**\( (t) \): \( t \) is free
- **Eternal**\( (t) \): \( t \) is eternal
In addition to the God predicate \( \text{God}(x_{\ell}) \), we also have the constant symbols \( \text{God}_{\ell} \), \( \text{God}_{\ell_{\text{th}}} \), \( \text{God}_{\ell_{\text{ex}}} \), ... for referring directly to God, both as an infinitely-faceted substance existing in the \( \mathbb{N} \)-domain of a world and as a single-faceted substance existing in each of its secondary domains.

Well-formed formulae of the formal Spinozian language are generated from its lexicon through the usual grammar. We interpret these formulae relative to a “pointed” Spinozian model (a Spinozian model \( M \) together with a world \( w \) drawn from the space \( W \) of this model) and to a variable assignment \( g \) that maps each variable of the language to some member of the corresponding global sortal domain(s)—where \( D^s \) is the union of all the global domains and \( \overline{D}^s \) is the union of only the global secondary domains, \( g(x_a) \in D_s \), \( g(x) \in D^s \), and \( g(\bar{x}) \in \overline{D}^s \). Our official definition of a Spinozian model integrates the intraworld structure from §2.1 with the interworld structure from §2.2 and adds a function for interpreting the nonlogical symbols in the language:

**Spinozian models**

A many-sorted Spinozian model \( M \) is an ordered tuple consisting of a nonempty set of worlds \( W \) with designated point \( @ \in W \), a domain assignment \( D_s \) for each sort \( s \in S_{\text{Spinoza}} \) mapping every world \( w \in W \) to a set of entities drawn from a global domain \( D_s \), a projection function \( \pi_s \) for each secondary sort \( s \in \{ \text{Ex, Th,} \ldots \} \), an essence function \( E \), a conceivability function \( C \), and—this is the new ingredient—an interpretation function \( I \) mapping each constant in the language to a member of the corresponding global domain and each \( n \)-adic nonlogical predicate symbol and world \( w \) to an \( n \)-ary relation over the global domains (i.e., a subset of the \( n \)-fold Cartesian product \( D^s \) of \( D^s \)):

a. For each constant \( c_s \), \( I(c_s) \in D_s \).

b. For each nonlogical predicate \( P(t_1, \ldots, t_n) \),
   \[ I(P(t_1, \ldots, t_n), w) \subseteq D^n. \]

We interpret constant and predicate symbols over the global domains of the model, and not just over the local domains of worlds. This keeps the model theory flexible: constants can refer to both existing and non-existing things at a world, and predicates can be instantiated by both existents and nonexistents (though we don’t need this flexibility for much of what follows). While constant symbols are rigid designators whose denotations cannot vary from world to world (as in Kripke 1980), the interpretation of predicates is world-dependent. That said, we assume throughout this paper that an element’s ontological status (i.e., whether it is a substance or mode, whether it is God, and so forth) is, like its sort and essence and conceivability sets, non-contingent.

With the information in Spinozian models and variable assignments, we can recursively evaluate each grammatical expression of the Spinozian
language. In the semantic clauses that follow, we use \( \varphi \) and \( \psi \) as schematic variables ranging over well-formed formulae, and we write \( \mathcal{M}, g, w \models \varphi \) to mean that \( \mathcal{M} \) satisfies \( \varphi \) at world \( w \) with assignment \( g \), and \( \mathcal{M}, g, w \not\models \varphi \) to mean that \( \mathcal{M} \) does not satisfy \( \varphi \) at \( w \) with \( g \).

The semantics for the non-modal fragment of the language is relatively standard. We first compute the extensions of terms in the usual way, where constants denote their interpretation in \( \mathcal{M} \) and variables denote the values assigned to them by the function \( g \):

**Term denotations**

The denotation \( [t]_{\mathcal{M},g,w} \) of term \( t \) at \( w \) with respect to \( \mathcal{M} \) and \( g \) is defined as follows:

a. \( [c]_{\mathcal{M},g,w} = I(c) \).

b. \( [x]_{\mathcal{M},g,w} = g(x) \), \( [\bar{x}]_{\mathcal{M},g,w} = g(\bar{x}) \).

We then compositionally assign satisfaction conditions to well-formed formulae using these denotations. Starting with atomic formulae, there are three cases to consider: predications, equations, and sortal-projective claims. To evaluate an \( n \)-adic nonlogical predicate symbol applied to \( n \) terms, we check to see whether the denotations of these terms stand in the \( n \)-ary relation expressed by the predicate:

**Interpretation of predications**

\( \mathcal{M}, g, w \models P(t_1, \ldots, t_n) \) iff \( \langle [t_1]_{\mathcal{M},g,w}, \ldots, [t_n]_{\mathcal{M},g,w} \rangle \in I(P(t_1, \ldots, t_n), w) \)

Standard, projective, and cross-attribute identity claims are evaluated by checking whether the denotations of terms on either side of the relevant identity symbol are identical in the senses discussed in §2.1:

**Interpretation of identity claims**

\( \mathcal{M}, g, w \models t = t' \) iff \( [t]_{\mathcal{M},g,w} = [t']_{\mathcal{M},g,w} \)

\( \mathcal{M}, g, w \models t_\approx = p \bar{t} \) iff \( [t_\approx]_{\mathcal{M},g,w} = p [\bar{t}]_{\mathcal{M},g,w} \)

\( \mathcal{M}, g, w \models \bar{t} = C P \bar{t} \) iff \( [\bar{t}]_{\mathcal{M},g,w} = C [P]_{\mathcal{M},g,w} \)

As for the sort predicates ‘Same-sort(\( t, t' \))’ and ‘All-sorts(\( t_\approx \))’, the former checks whether its arguments are of the same sort while the latter checks whether the multifaceted entity denoted by its argument is projected into each of the infinitely many secondary sortal domains:

**Interpretation of sortal-projective claims**

\( \mathcal{M}, g, w \models \text{Same-sort}(t, t') \) iff \( [t]_{\mathcal{M},g,w} \in D_s \) iff \( [t']_{\mathcal{M},g,w} \in D_s \) for each sort \( s \in Spinoza \)

\( \mathcal{M}, g, w \models \text{All-sorts}(t_\approx) \) iff \( \pi_s(w)([t_\approx]_{\mathcal{M},g,w}) \) is defined for each secondary sort \( s \in \{ \text{Th}, \text{Ex}, \ldots \} \)

Moving on to the sentential connectives, we assume that they have the classical semantics:
Interpretation of sentential connectives

\[ M, g, w \models \neg \varphi \iff M, g, w \not\models \varphi \]
\[ M, g, w \models \varphi \land \psi \iff M, g, w \models \varphi \text{ and } M, g, w \models \psi \]
\[ M, g, w \models \varphi \lor \psi \iff M, g, w \models \varphi \text{ or } M, g, w \models \psi \]
\[ M, g, w \models \varphi \rightarrow \psi \iff M, g, w \not\models \varphi \text{ or } M, g, w \models \psi \]
\[ M, g, w \models \varphi \equiv \psi \iff M, g, w \models \varphi \rightarrow \psi \text{ and } M, g, w \models \psi \rightarrow \varphi \]

We also give a standard treatment of universal/existential quantification, though we have a range of quantificational options corresponding to the different quantificational domains available in our models (reflected in the availability of both actualist and possibilist quantifiers and the range of variable types in the language). Starting with actualist quantification over specific local secondary domains and letting \( g_{[x_s \rightarrow a_s]} \) be the variant assignment that is exactly like the variable assignment \( g \) except it sends the variable \( x_s \) to \( a_s \), we evaluate quantified statements of the form ‘\( \forall x_s \varphi \)’ and ‘\( \exists x_s \varphi \)’ as follows:

**Interpretation of actualist quantifiers**

\[ M, g, w \models \forall x_s \varphi \iff \text{for all } a_s \in D_s(w), M, g_{[x_s \rightarrow a_s]}, w \models \varphi \]
\[ M, g, w \models \exists x_s \varphi \iff \text{for some } a_s \in D_s(w), M, g_{[x_s \rightarrow a_s]}, w \models \varphi \]

Informally, ‘\( \exists x_s \varphi \)’ is satisfied by \( M \) at \( w \) with \( g \) iff there is some way of keeping the values of all other variables fixed but assigning some existent in the local domain \( D_s(w) \) to \( x_s \) to satisfy the embedded formula \( \varphi \) with that variant assignment of variables. ‘\( \forall x_s \varphi \)’ is satisfied by \( M \) at \( w \) with \( g \) iff no matter how \( g \) is varied on \( x_s \) the result satisfies \( \varphi \). Actualist quantification with unindexed variables is analogous—where \( D^*(w) \) is the union of all the local domains of \( w \) (i.e., the set of all existents of any sort in \( w \)) and \( D^*(w) \) is the union of only the local secondary domains (i.e., the set of all existents of any secondary sort in \( w \)), we give the following additional entries:

**Interpretation of actualist quantifiers (continued)**

\[ M, g, w \models \forall x \varphi \iff \text{for all } a \in D^*(w), M, g_{[x \rightarrow a]}, w \models \varphi \]
\[ M, g, w \models \exists x \varphi \iff \text{for some } a \in D^*(w), M, g_{[x \rightarrow a]}, w \models \varphi \]
\[ M, g, w \models \forall \bar{x} \varphi \iff \text{for all } \bar{a} \in D^*(w), M, g_{[\bar{x} \rightarrow \bar{a}]}, w \models \varphi \]
\[ M, g, w \models \exists \bar{x} \varphi \iff \text{for some } \bar{a} \in D^*(w), M, g_{[\bar{x} \rightarrow \bar{a}]}, w \models \varphi \]

We also allow for possibilist quantification over the global domains of a model. Whereas ‘\( \forall \)’ and ‘\( \exists \)’ quantify over only the existing things in a world, the possibilist universal and existential quantifiers ‘\( \Pi \)’ and ‘\( \Sigma \)’ quantify over all things, whether they exist or not, and whether they are conceivable at the world of evaluation or not. We provide the following clauses for general quantified statements of the form ‘\( \Pi x \varphi \)’ and ‘\( \Sigma x \varphi \)’ (the remaining cases are similar):

**Interpretation of possibilist quantifiers**

\[ M, g, w \models \Pi x \varphi \iff \text{for all } a \in D^*, M, g_{[x \rightarrow a]}, w \models \varphi \]
\[ M, g, w \models \Sigma x \varphi \iff \text{for some } a \in D^*, M, g_{[x \rightarrow a]}, w \models \varphi \]
In some parts of the *Ethics*, Spinoza clearly has actualist quantification in mind. In others, he needs possibilist quantification. In still others it is unclear what he intends to quantify over. In this paper, we adopt a conservative methodology and try to get by with actualist quantification as much as possible. We introduce possibilist quantification only when it is absolutely required (starting with our treatment of E1p11d2).

The remaining entries are for the modal operators whose semantics involves the essence and conceivability functions. First, the essence of a thing necessitates that \( \varphi \) iff the proposition expressed by \( \varphi \) with respect to \( \mathcal{M} \) and \( g \) (i.e., the set of worlds \( \{ w : \mathcal{M}, g, w \models \varphi \} \)) lies in the essence function \( E \) applied to this thing:

**Interpretation of necessary-by-essence operators**
\[
\mathcal{M}, g, w \models \square \varphi \text{ iff } \{ w : \mathcal{M}, g, w \models \varphi \} \in E(\mathcal{E}[\mathcal{M}, g, w])
\]

Metaphysical necessity/possibility is necessity/possibility relative to the essences of all members of the global domains of the model:

**Interpretation of metaphysical modality operators**
\[
\begin{align*}
\mathcal{M}, g, w \models \Box \varphi & \iff \text{ for all } v \in W \text{ s.t. } v \in \bigcap \mathcal{E}(a) \text{ for each } a \in \mathcal{D}^*, \\
\mathcal{M}, g, v \models \varphi \\
\mathcal{M}, g, w \models \Diamond \varphi & \iff \text{ for some } v \in W \text{ s.t. } v \in \bigcap \mathcal{E}(a) \text{ for each } a \in \mathcal{D}^*, \\
\mathcal{M}, g, v \models \varphi
\end{align*}
\]

Finally, it is conceivable about a thing that \( \varphi \) when attending to only its essence iff the proposition expressed by \( \varphi \) with respect to \( \mathcal{M} \) and \( g \) lies in the conceivability set of this thing:

**Interpretation of conceivability operators**
\[
\mathcal{M}, g, w \models \Diamond \varphi \text{ iff } \{ w : \mathcal{M}, g, w \models \varphi \} \in C(\mathcal{C}[\mathcal{M}, g, w])
\]

Having recursively assigned satisfaction conditions to every formulae of the Spinozian language, we next define truth for its *sentences*—formulae with no free variables, whose satisfaction condition does not depend on the particular variable assignment \( g \) used—in the usual way, where \( \varphi \) is true in the pointed model \( \mathcal{M}, w \) (notation: \( \mathcal{M}, w \models \varphi \)) just in case \( \mathcal{M} \) satisfies \( \varphi \) at \( w \) with any assignment:

**Sentential truth**

Where \( \varphi \) is a sentence, \( \mathcal{M}, w \models \varphi \) iff \( \mathcal{M}, g, w \models \varphi \) for any \( g \).

We identify valid arguments as those that preserve truth in all models:

**Logical validity**
\[
\varphi_1, ..., \varphi_n \models \psi \text{ iff } \text{ there is no model } \mathcal{M} \text{ and world } w \text{ such that } \mathcal{M}, w \models \varphi_1, ..., \mathcal{M}, w \models \varphi_n, \text{ but } \mathcal{M}, w \not\models \psi.
\]

In words: the argument from premises \( \varphi_1, ..., \varphi_n \) to conclusion \( \psi \) is valid iff there is no pointed model \( \mathcal{M}, w \) such that \( \varphi_1, ..., \varphi_n \) are all true in this pointed model but \( \psi \) is not.
3 Definitions

With our Spinozian model theory in place, we can finally get down to work reconstructing the *Ethics*, beginning with its opening definitions E1d1-E1d8. Importantly, for Spinoza, as for many medieval and early modern philosophers operating with a notion of *real definition*, defining is no easy task—a good definition cannot simply state the meaning of a new expression in terms of previously understood expressions (as on the modern conception of a definition) but must also capture the *essence* of what is defined. Spinoza writes in the *Treatise on the Emendation of the Intellect*,

> To be called perfect, a definition will have to explain the inmost essence of a thing, and to take care not to use certain *propria* in its place. (TdIE §95)

He also writes in E1p8s2,

> The true definition of each thing neither involves nor expresses anything except the nature of the thing defined. (cf. Ep. 34 (IV/179/17)).

Moreover, Spinoza requires of a definition that it suffice for deducing all the properties of the thing defined:

> We require a concept, or definition, of the thing such that when it is considered alone, without any others conjoined, all the thing’s properties [*proprietas*] can be deduced from it. (TdIE §96)

So, if Spinoza has done his job well, his lead-off definitions express the essences of his fundamental metaphysical ingredients and can serve as a platform for deducing their properties.

The text begins with the definition of a *cause of itself* [*causa sui*]:

D1: By cause of itself I understand that whose essence involves existence, or that whose nature cannot be conceived except as existing [*Per causam sui intelligo id, cujus essentia involvit existentiam, sive id, cujus natura non potest concipi, nisi existens*].

The first part of this definition characterizes a cause of itself in terms of what essence involves. This can be formalized as follows, where the existence predicate ‘*Exists*(*t*)’ abbreviates ‘∃*x*(x = *t*)’:

\[
\text{E1d1}_*. \forall *x*(\text{Causa-sui}(*x*)) \equiv_d \Box_x \text{Exists}(*x*)^31
\]

---

31One might reasonably expect a cause of itself to also be *⇝*-related to itself, where ‘*⇝*’ is the binary causal relation symbol in the Spinozian language. We make this connection explicit in E1p7d when it is needed (and see n. 64 for a more precise formulation).
The “or” in E1d1 translates the Latin “sive”, which introduces a second characterization of a *causa sui* in terms of conceivability (E1d1 shouldn’t be regarded as a logical disjunction), and we translate it as follows:

\[
E1d1.b. \forall x (\text{Causa-sui}(x) \equiv_{df} \neg \Box_x \neg \exists x) 
\]

In words: a thing is a *causa sui* iff it is inconceivable about this thing attending only to its nature (essence) that it fails to exist. Stringing together E1d1.a and E1d1.b generates the following equivalence:

\[
\forall x (\Box_x \exists x \equiv \neg \Box_x \neg \exists x) 
\]

The left-to-right direction is E1a7, an instance of the general principle that what is conceivable about a thing cannot conflict with its essence.\(^{32}\)

Spinoza’s second definition in Part One concerns *finitude*:

D2: That thing is said to be finite in its own kind that can be limited by another of the same nature.

Note the modal element in this definition: E1d2 requires that a thing finite in its own kind *can* be limited [terminari potest] by another of the same nature, not that there must actually exist something of the same nature that limits it. What does Spinoza mean by “of the same nature”? In an ancillary passage of E1d2, Spinoza tells us that a body (i.e., a mode of Extension) would have to be limited by another body, and a thought (i.e., a mode of Thought) by another thought. He also claims later in E1p8d that a finite substance would (per impossible) have to be limited by another substance. Thus, on our interpretation, the conceived limiter must be of the same sort (extended/thinking/etc.) and ontological kind (substance or mode) as the finite limited thing. Formalizing this reading, we translate E1d2 as follows:

\[
E1d2. \forall \bar{x} (\text{Finite-in-kind}(\bar{x}) \equiv_{df} \diamond \exists \bar{y} (\bar{x} \neq \bar{y} \land \bar{x} \dashv \bar{y} \land \text{Same-sort}(\bar{x}, \bar{y}) \land \\
(\text{Substance}(\bar{x}) \equiv \text{Substance}(\bar{y})) \land \\
(\text{Mode}(\bar{x}) \equiv \text{Mode}(\bar{y})))) 
\]

In words: a thing is finite in its own kind iff it is possible that there exists another thing that limits it of the same sort and ontological kind.\(^{33}\)

\(^{32}\)On another way of reading the material after “or” in E1d1, Spinoza is saying that whether something counts as a cause of itself depends not on what is conceivable about this thing attending to only its nature but rather on what is conceivable *about its nature*. We might alternatively translate the second half of E1d1 as follows:

\[
E1d1.b'. \forall x (\text{Causa-sui}(x) \equiv_{df} \neg \Box_x \neg \exists x) 
\]

In words: a thing is a *causa sui* iff it is inconceivable that its essence doesn’t involve existence. While this translation using nested modals is more complex than E1d1.b, it has the virtue of not rendering E1a7 redundant. In any case, in the demonstrations of the *Ethics* where E1d1 is invoked (E1p7d, E1p24d, and E5p35d), only the first non-\(\Box\)-characterization is used, so getting the second \(\Box\)-characterization exactly right isn’t required for reconstructing any of the argumentative structure of the text.

\(^{33}\)Shouldn’t we say that a thing \(a\) is finite in its own kind in \(w\) iff it is possible, holding fixed its sort, ontological kind, and ‘size’ in \(w\), that it is limited by another of
In the next four definitions, Spinoza lays out the basic building blocks of his metaphysical system. He begins with *substance*:

**D3**: By substance I understand what is in itself and is conceived through itself, i.e., that whose concept does not require the concept of another thing, from which it must be formed.

Sticking close to the conjunctive surface form of E1d3 preceding the “i.e. *hoc est*”, we first offer this translation:

**E1d3.** $\forall x (\text{Substance}(x) \equiv_d x \subseteq x \land x \subseteq x)$

However, in E1p1d and E1p2d, among other places, Spinoza requires the following strengthened version of this definition, which we call E1d3* (in the demonstrations that follow, we let Spinoza help himself to both E1d3 and E1d3*):

**E1d3*. $\forall x (\text{Substance}(x) \equiv_d x \subseteq x \land \forall y (x \subseteq y \rightarrow x = y) \land x = x \land \forall y (x \subseteq y \rightarrow x = y)$

In words: substance is in *only* itself and is conceived through *only* itself. Arguably, the strengthened $\subseteq$-condition in E1d3* captures the material following the “i.e.” in Spinoza’s definition of substance. If $x \subseteq y$ can be paraphrased as *the concept of x requires the concept of y for its formation* (as suggested in various places in the *Ethics*, such as E1a5, E1p8s2, and E1p15d), then the strengthened $\subseteq$-condition says that the concept of a substance does not require the concept of another thing for its formation.

Spinoza next defines *attribute*, which is one of the most exegetically challenging passages in the entire *Ethics*:

**D4**: By attribute I understand what the intellect perceives of a substance, as constituting its essence.

Spinoza’s attributes led us to develop a two-tiered many-sorted model theory where the projection of a multifaceted entity from the primary domain of a world into one of its secondary domains is this very same entity perceived by the intellect as having existence of a particular kind. Within this formal architecture, we can give a fairly simple definition of an attribute as the projection of a multifaceted substance (more on the phrase “as constituting its essence” in E1d4 in a moment):

**E1d4.** $\forall \bar{x} (\text{Attribute}(\bar{x}) \equiv_d \exists x_{\mathcal{K}} (\text{Substance}(x_{\mathcal{K}}) \land x_{\mathcal{K}} =_{p} \bar{x})$)

In the world $@$ according to Spinoza, the attribute of Extension is the extended substance $\pi_{E\mathcal{K}}(\mathcal{K}@)(\text{God}_\mathcal{K})$ (from E1p15s: “extended substance the same nature? Yes, but this raises the thorny problem of cross-world predication (see Kocurek 2016 for recent discussion). To circumvent this, we just implicitly assume in this paper that the relative ‘size’ of things does not vary across worlds in a model.
is one of God’s infinite attributes”), the attribute of Thought is the thinking substance π_{Th}(\text{God}_A), and so forth.

One might object to this “substantival interpretation” of Spinoza’s attributes (as Lin 2006 calls it) on the grounds that it contradicts his substance monism:

If God exists and God is a being with infinitely many attributes and every attribute is a substance, then, far from being the only substance, God must be only one substance among infinitely many others. (Lin 2006, p. 148)

However, we disagree that our treatment of attributes commits us to the anti-Spinozian doctrine that there is in reality more than one substance. On the interpretation we have been developing, God—a multifaceted substance—is projectively identical to each of its attributes, which in turn are cross-attributively identical to each other, but these attributes can still be distinguished by the standard identity relation—the more fine-grained identity relation that counts things as distinct when they have different kinds of existence—because they are God perceived by the intellect as existence of different kinds.\(^{34}\)

Another possible objection to our substantival interpretation is that we are committing a category mistake by even treating attributes as elements in Spinoza’s ontology. Like us, Garrett (2017) takes Spinoza to be strongly committed to the doctrine of “Substance-Attribute Identity” on which the attribute of Thought is the thinking substance and the attribute of Extension is the extended substance. However, Garrett doesn’t think that Substance-Attribute Identity should be understood as a doctrine concerning the identity of items within Spinoza’s ontology, because attributes are neither substances nor modes (which exhaust the ontological options per E1p4d) but serve only to structure Spinoza’s ontology:\(^{35}\)

As manners of existence, attributes are not elements in Spinoza’s ontology but rather structures of his ontology....Spinoza considers attributes not as substance or modes but rather as manners in which—or “through which” or “under which,” as we might also say—substance and modes exist. (p. 29)

\(^{34}\)Jarrett (1978) also defends a substantival view of attributes from the objection that it contradicts substance monism. His solution (which is related to our own) is to dispense with “absolute unqualified identity” altogether and replace it with the primitive relations is the same substance as and is the same attribute is.

\(^{35}\)As a result, Garrett must tell a relatively complicated story to make sense of Spinoza’s commitment to Substance-Attribute Identity involving the “Adequate-Idea Conception of Truth” (see his paper for details). In contrast, one virtue of our account is that we can regard a sentence like ‘The attribute of Thought is the thinking substance’ as a straightforward standard identity claim.
In our model-theoretic setup, one could spell out Garrett’s proposal by interpreting attributes not as elements in the secondary sortal domains of a world but rather as these sortal domains themselves—indeed, Garrett sometimes speaks of attributes as “domains of existence”. For instance, a Garrett-style account might identify the attribute of Thought not with the thinking substance $\pi_{\text{Th}}(\underline{a})(\text{God})_\mathbb{R}$ but rather with the entire domain $D_{\text{Th}}(\underline{a})$ of things existing as thinking, which includes but isn’t identical to the thinking substance.

Now, it isn’t clear how much really turns on this choice point in the context of Spinoza’s broader metaphysics. Given that the thinking substance anchors the entire domain of thinking things in the sense that every element in $D_{\text{Th}}(\underline{a})$ inheres in and is conceived through this unique substance, the distinction between element and structure is somewhat blurred when it comes to substance. That said, there is considerable textual support in the Ethics for identifying Spinoza’s attributes with substance—in effect, for treating each attribute as an element in his ontology, albeit one that also serves to structure the ontology. Spinoza says in E1p14c2 that “an extended thing [res extensa] and a thinking thing [res cogitans] are either attributes of God, or affections of God’s attributes” (our emphasis). He also explicitly equates substances with their attributes in E1p4d, and he writes in E1p28d that “modes are nothing but affections of God’s attributes”, which echoes his definition of mode (E1d5) except that “God’s attributes” replaces “a substance”. At various places in the Ethics, Spinoza also ascribes to attributes many of the characteristic features of substance. For instance, he proves that attributes are conceived through themselves (E1p10), eternal (E1p19), and immutable (E1p20c2). In E1p29s (II/71/9-11), he also asserts that attributes are in themselves. Moreover, Spinoza thinks that attributes are causally efficacious (E1p21), which is especially puzzling if they are non-elements because we would then need to understand causality as a cross-categorical relation applying both to elements and to non-elements.

In sum, we think that our substantival interpretation can be defended against both objections raised. Moving on, then, why does Spinoza say in E1d4 that attributes are what the intellect perceives of substance “as constituting its essence”? What exactly is the essence of substance? If you recall, Spinoza’s definitions are intended to capture the inner-most essences of the things defined. E1d3 in particular tells us that the essence of substance is that substance is in itself and conceived through itself, and E1d4 then tells us that an attribute is substance perceived by the intellect as something in itself and conceived through itself. To capture the full content of E1d4, we assume that this definition must be supplemented with the following constraint:

Projection Constraint:
$$\forall x_N \forall \bar{x}(x_N =_p \bar{x} \rightarrow (\text{Substance}(x_N) \equiv \text{Substance}(\bar{x})))$$
Combined with our formal translation of E1d4 above, this ensures that an attribute \( \bar{a} \) has the ontological and conceptual independence essential to substance. Plugging in E1d3, for instance, returns the following:
\[
\forall \bar{x}(\text{Attribute}(\bar{x}) \equiv \bar{x} \leq \bar{x} \land \bar{x} \sqsubseteq \bar{x}) \quad (\text{see E1p29s and E1p10})
\]

Alternatively, we can use the strengthened definition E1d3* to generate the condition that an attribute inheres in only itself and is conceived through only itself. Now, if you also recall, Spinoza requires of a true or perfect definition that it suffice for deducing all the properties of the thing defined. So, the essence of substance suffices for deducing all its properties, and each attribute of a substance offers a window or portal through which one can fully understand this substance \emph{qua substance of a certain kind} (see E2p6d). For Spinoza, we can come to know everything about God \emph{qua substance} by considering any attribute (per E1p10s, each attribute “expresses the reality, or being of substance”). That said, there remain things we cannot come to know about God from this vantage, such as that it has infinitely many attributes.

In his fifth definition, Spinoza defines \emph{mode}:

\[\text{D5: By mode I understand the affections of a substance, or that which is in another through which it is also conceived.}\]

We translate the material preceding the “or” (again translating the Latin “\emph{sive}”) as follows:

\[\text{E1d5}_a. \forall x(\text{Mode}(x) \equiv df \exists y(\text{Substance}(y) \land \text{Affection}(x, y)))\]

The rest of the definition might be taken to complete E1d5\(_a\) by spelling out what it means for one thing to be an affection of another:

\[\text{E1d5}_a(\text{cont}). \forall x \forall y(\text{Affection}(x, y) \equiv df x \neq y \land x \leq y \land x \sqsubseteq y)\]

However, this cannot be all there is to Spinoza’s definition of mode. In the material following “or”, we take Spinoza to be providing a second alternative characterization of a mode (in addition to indicating how he understands affection-hood as per our continuation of E1d5\(_a\)):

\[\text{E1d5}_b. \forall x(\text{Mode}(x) \equiv df \exists y(x \neq y \land x \leq y \land x \sqsubseteq y))\]

This second characterization could seem too permissive, as it recognizes as a mode whatever is in another, even if it isn’t in a substance. If there

\[\text{In his landmark study, \textit{Spinoza’s Metaphysics}, Curley (1969) challenges the common perception of Spinozian modes, arguing that Spinoza is using the term “mode” in an idiosyncratic sense: Spinoza’s modes are neither states nor qualities of substance, but merely effects of substance. While this thesis was a catalyst for the study of Spinoza in the Anglo-American world, there is nonetheless an overwhelming body of textual evidence disproving Curley’s reading (such as the very definition of a mode as an \textit{affection} of substance), and we interpret Spinoza’s “mode” in the sense used by his contemporaries and interlocutors. For extensive critiques of Curley’s reading, see Carriero (1995) and Melamed (2013), pp. 3–60.}\]
is an infinite chain of entities, each inhering in and conceived through the next, that never bottoms out in a substance, then the members of this chain count as modes on E1d5b but not on E1d5a. But Spinoza infers in E1p4d from the fact that something is in another that this thing is an affection of substance. For this step of the demonstration to go through, he must regard the two characterizations of a mode in E1d5a and E1d5b as equivalent. Note that when E1d5 is interpreted in this way as offering two equivalent characterizations of mode, it packs in the thick metaphysical commitment that if something is an affection of something else then it is an affection of substance.

Spinoza next defines *God*:

D6: By God I understand a being absolutely infinite, i.e., a substance consisting of an infinity of attributes, of which each one expresses an eternal and infinite essence.

The part of E1d6 before the “i.e. *hoc est*” can be translated as follows:

\[
\forall x (\text{God}(x) \equiv_d \text{Abs-infinite}(x))
\]

The rest of the definition unpacks what it is to be a “being absolutely infinite”. The first characteristic mark of an absolutely infinite being is that it is a substance. The second is that it consists of an “infinity of attributes”. As we understand this, God must be projected into all of the infinitely many domains of quantification corresponding to the secondary sorts \{Th, Ex, ...\} (it doesn’t suffice for absolute infinitude to be projected into a proper infinite subset of these secondary domains). Spinoza says this explicitly in E1p14d:

God is an absolutely infinite being, of whom no attribute which expresses an essence of substance can be denied.

In E1d6 and E1p14d, Spinoza also says that each attribute constituting God “expresses *exprimit*” an eternal and infinite essence of substance. Spinoza’s concept of expression, the first appearance of which in the *Ethics* is in E1d6, has been the subject of increased attention in recent Spinoza scholarship and requires more careful discussion than we can offer here (see especially Della Rocca 2002; Gartenberg 2017; Morrison *ms.*). We want to remain fairly neutral on this issue, so we assume only that, at least as Spinoza uses *exprimit* in E1d6 and E1p14d, for an attribute to express an eternal and infinite essence of substance entails that it follows from the essence of this attribute that it is infinite in its own kind—the notion of infinity appropriate to unifaceted entities—and eternal. Indeed, Spinoza proves in E1p19 that all of God’s attributes are eternal, and he proves in E1p8 that each attribute is necessarily infinite in its own kind (as we interpret this proposition). Summing

\[37\text{Thanks to Filip Geaman for pointing this out.}\]
up, we complete our treatment of E1d6 as follows, where ‘\(\text{Inf-in-kind}(\bar{t})\)’ abbreviates ‘\(\neg\text{Finite-in-kind}(\bar{t})\)’:

\[ \text{E1d6 (cont). } \forall x_{\aleph}(\text{Abs-infinite}(x_{\aleph}) \equiv_{df} \text{Substance}(x_{\aleph}) \land \text{All-Sorts}(x_{\aleph}) \land \forall \bar{x}(x_{\aleph} =_{p} \bar{x} \rightarrow \Box_{\bar{x}}(\text{Eternal}(\bar{x}) \land \text{Inf-in-kind}(\bar{x}))) \]

In words: a multifaceted thing is absolutely infinite (equivalently, God) iff this thing is a substance projected into each of the infinitely many secondary sortal domains and it follows from the essence of each of its projections (i.e., each of its attributes) that this projection is eternal and infinite in its own kind.

The two remaining definitions in Part One for freedom and eternity aren’t explicitly used in the fragment of the *Ethics* under investigation. While we provide formal treatments of E1d7 and E1d8 for the sake of completeness, our discussion of these definitions will be brief. Here is the definition of *freedom*:

D7: That thing is called free which exists from the necessity of its nature alone, and is determined to act by itself alone.
But a thing is called necessary, or rather compelled, which is determined by another to exist and to produce an effect in a certain and determinate manner.

We translate the first sentence of E1d7 as follows:

\[ \text{E1d7. } \forall x(\text{Free}(x) \equiv_{df} \Box_{x}(\text{Exists}(x)) \land \forall y(\Box y(\text{Exists}(x)) \rightarrow x = y) \land \forall y(x \rightsquigarrow y \rightarrow (\Box_{z}(x \rightsquigarrow y) \land \forall z(\Box_{z}(x \rightsquigarrow y) \rightarrow x = z))) \]

Our formulation captures the core of E1d7—that is, defining freedom as existing and being determined to act by virtue of one’s essence alone. The first two lines of our formula capture the requirement to *exist* from one’s essence alone while the latter two capture the requirement to *be determined to act* by virtue of one’s essence alone. In the second sentence of E1d7, Spinoza clarifies that freedom is contrasted not with necessity (as all things are necessary for Spinoza) but rather with compulsion—being necessitated by (the nature) of another thing.

The final definition of Part One is for *eternity*:

D8: By eternity I understand existence itself, insofar as it is conceived to follow necessarily from the definition alone of the eternal thing.

For Spinoza, eternity is not existence in all times (see E1d8e). In fact, Spinoza seems to think of eternity as a modal rather than temporal notion. Eternity is the existence of that which exists necessarily, solely by virtue of its own essence. Thus, our translation is this:
Notice that the content of E1d1, E1d7, and E1d8 is very similar. Indeed, on our translations, a thing is free only if it is eternal, and eternal only if it is a cause of itself.

4 Axioms

Spinoza thinks of axioms as general (TIE §93), evident truths (E1p8s2 (II/50/3) and Ep. 56 (IV/260/28)) requiring no demonstration (DPP I/201/22), though he sometimes suggests that an axiom follows from a definition (Ep. 4 (IV/13/32)) or even from a previously established proposition (E5a2). While both axioms and definitions are “eternal truths”, axioms are unlike definitions in that they are not intended to spell out the essences of things (Ep. 9 (IV/43/33)).

Spinoza’s first axiom in Part One effectively tells us that everything inheres in something:

A1: Whatever is, is either in itself or in another.

The translation is straightforward:

E1a1. \( \forall x (x \subseteq x \lor \exists y (x \subseteq y \land x \neq y)) \)

In E1a1, the “or” translates the Latin “\( \text{vel} \)”, which is an inclusive logical disjunction. So, E1a1 alone doesn’t rule out the possibility of a thing inhering both in itself and in another. But this possibility is excluded by the combination of E1d3 and E1d3*—if a thing inheres in itself, it is a substance by E1d3, and so inheres in only itself by E1d3*.

Spinoza’s second axiom is the \( \subseteq \)-equivalent of E1a1:

A2: What cannot be conceived through another, must be conceived through itself.

The translation parallels that of E1a1:

E1a2. \( \forall x (\neg \exists y (x \subseteq y \land x \neq y) \rightarrow x \subseteq x) \)

Equivalently: \( \forall x (x \subseteq x \lor \exists y (x \subseteq y \land x \neq y)) \)

This conceptual axiom tells us that everything is conceivable through something. Since for Spinoza “conceived through” implies “is explained through” (see Della Rocca 1996, 3-4), E1a2 commits Spinoza to the claim that everything is explainable by, or in terms of, something. Unlike E1a1 as deployed in E1p4, E1p6c, E1p11, E1p14cs, E1p15, and E1p28, E1a2 isn’t explicitly used in any of the demonstrations of the Ethics. However, some of the argumentation using E1a1 could equally well have proceeded from E1a2, such as Spinoza’s argument that everything is either a substance or mode in E1p4d.
Spinoza’s third axiom isn’t used in the initial fragment of the *Ethics* under investigation either, but we treat it for completeness:

A3: From a given determinate cause the effect follows necessarily; and conversely, if there is no determinate cause, it is impossible for an effect to follow.

We translate E1a3 as follows:

\[ \forall x \forall y (x \leadsto y \rightarrow \Box (\text{Exists}(x) \equiv (\text{Exists}(y) \land x \leadsto y))) \]

In words: if a thing causes another then necessarily if the cause exists it causes the effect (which also exists), and if the cause doesn’t exist then it isn’t the case that the effect exists and is caused by the cause.

Next is Spinoza’s “causal axiom”:

A4: The cognition of an effect depends on, and involves, the cognition of its cause [Effectus cognitio a cognitione causae dependet et eandem involvit].

Given the way this axiom is formulated and how it is used in E1p3d and E1p6d, we take it to convey that causation implies conception:

\[ \forall x \forall y (x \leadsto y \rightarrow y \subseteq x) \]

The causal axiom E1a4 is widely thought to also convey the converse implication from conception to causation, given how Spinoza deploys this axiom in E1p25d (Jarrett 1978; Bennett 1984; Wilson 1991; Della Rocca 1996; Lin 2007; Garrett 2008; Melamed 2012b). However, the converse is controversial (see Morrison 2013 for a reconstruction of E1p25d on which only the implication from causation to conception is needed), and in any case we do not need to translate E1a4 using a biconditional to reconstruct any of the demonstrations we focus on in this paper. So we work with only the causation \( \rightarrow \) conception direction, though we want to remain officially neutral about whether E1a4 also carries commitment to the controversial converse.

Here is Spinoza’s fifth axiom:

A5: Things that have nothing in common with one another also cannot be understood through one another, or the concept of the one does not involve the concept of the other.

We translate E1a5 as follows:

\[ \forall x \forall y (x \perp y \rightarrow (x \not\subseteq y \land y \not\subseteq x)) \]

This axiom includes the first occurrence of the phrase “have nothing in common with one another [quaer nihil commune cum se invicem habent].”

\[^{38}\text{We depart slightly from Curley in translating “cognitio” as ‘cognition’ rather than ‘knowledge’ since, for Spinoza, “cognitio” may well be inadequate and false, whereas knowledge is factive (as in Morrison 2013, 2015).}\]
in the *Ethics*, which has drawn hardly any attention in the existing scholarship despite it being highly unclear what Spinoza means by it. We return to this issue when discussing Spinoza’s argument in E1p2d that two substances have nothing in common.

Here is Spinoza’s sixth axiom:

\[ A6: \text{A true idea must agree [debet convenire] with its object.} \]

We translate this axiom as follows:

\[ E1a6. \forall x (\text{True-idea}(x) \equiv \forall y (\text{Object}(y, x) \rightarrow \text{Agree}(x, y))) \]

In the part of the *Ethics* under consideration, E1a6 appears only in a brief parenthetical remark in E1p5d. We delay further discussion of this axiom until then.

The final axiom of Part One is closely related to E1d1 in telling us about things that are *not* causes of themselves:

\[ A7: \text{If a thing can be conceived as not existing, its essence does not involve existence.} \]

In fact, on our translation E1a7 follows from E1d1:

\[ E1a7. \forall x (\Diamond_x \neg \exists x \rightarrow \neg \Box_x \exists x) \]

While Spinoza regards axioms as evident general truths requiring no demonstration, he seems to allow for axioms to follow from definitions in Ep. 4 (IV/13/32) (see also our n. 32 for an alternative reading of E1d1 on which E1a7 isn’t redundant).

Before moving on, we need to clarify the status of definitions and axioms in our model theory. It might be tempting to treat E1d1-E1d8 and E1a1-E1a7 as absolutely necessary in the sense that they hold at each world in a model for the *Ethics*. After all, Spinoza thinks that definitions and axioms are metaphysically necessary truths that cannot conflict with the essences of the things they are about. However, in addition to metaphysically possible worlds, Spinozian models include metaphysically impossible worlds that are incompatible with essences. In these impossible worlds, there is no guarantee that a definition or axiom holds. So, while we insist in what follows that Spinoza’s definitions and axioms are metaphysical necessities applying at each possible world, including the actual world @, we do not assume that his definitions and axioms hold at every world in a model.\(^{39}\)

\[ \text{5 Propositions} \]

In the first fifteen propositions of Part One, Spinoza develops his account of the unique, all-encompassing, infinitely-faceted divine substance. On

\(^{39}\)Spinoza’s propositions can also be regarded as metaphysically necessary, at least insofar as any tacit premises used in their proofs are metaphysically necessary.
our interpretation, one of the keys to unlocking this difficult portion of
the text is recognizing that Spinoza shifts between different perspectives
on his pluralistic ontology throughout the course of his argumentation.
Through the first eight propositions of the *Ethics*, Spinoza considers
single-faceted substances having existence of this or that kind. Only after
establishing that such substances are causes of themselves and infinite in
their own kind does Spinoza pan out to the ℵ-perspective and consider
the infinitely-faceted divine substance in its full glory, proving (among
other things) that it exists and everything inheres in it.

Our interpretation resembles that of Gueroult (1968), who famously
(or infamously) argued that Spinoza is referring throughout E1p1-E1p8
to substances consisting of, or constituted by, only a single attribute.
On Gueroult's reading, Spinoza *constructs* the essence of God in two
successive stages. First, Spinoza deduces the elements of this essence
(i.e., the one-attribute substances) in E1p1-E1p8. Then, he *integrates*
these simple elements into God's complete essence in E1p9-E1p15: “God
must be constructed from its elements just as the triangle is constructed
from its parts” (Gueroult 1968, p. 108). While this particular part of
Gueroult’s otherwise-celebrated work on Spinoza has some contemporary
supporters (Loeb 1981; Marshall 2009; Smith 2014), it has been widely
criticized (by Bennett 1984; Woolhouse 1993, among others) or simply
ignored in subsequent work on the early *Ethics* (Garrett 1990; Nadler
2006; Della Rocca 2008). So, before turning to Spinoza’s propositions
themselves we first want to clarify how our own Gueroult-like reading
of the text differs from Gueroult’s actual proposal, and discuss some
considerations in favor of our reading.

5.1 How to Have Your Gueroult and Eat It Too

There are two ways to understand the claim that Spinoza is trafficking in
“single-attribute substances” in the first eight propositions of the *Ethics*.
The first construal, which we attribute to Gueroult, is that Spinoza is
considering substances that have existence of *only* one kind (Gueroult
1968, pp. 107–140)—there isn’t yet a being exhibiting ontological plural-
ism in this part of the text. Instead, we have a manifold or “plurality of
substances constituted each by one attribute, incommensurable, unique
in their kinds, really distinct, existing by virtue of themselves, causes of
themselves, and infinite” (Gueroult 1968, p. 141). When the substances
of E1p1-E1p8 are thought of this way, there is a real puzzle about how
they can come together into a single unified divine substance consisting
of infinite attributes (Donagan 1991; Woolhouse 1993). Indeed, the very
possibility of the integration or construction of God from infinitely many
single-attribute substances seems to be precluded by Spinoza’s insistence
that infinity *cannot* be composed from finite parts (KV I, i (I/18/10) and
E1p15s (II/58/27)), and that there is no proportion between the finite
and the infinite (Ep. 54 (IV/253/7-11)). Moreover, given that Spinoza
has already introduced the possibility of an infinitely-faceted substance
in his definition of God (E1d6), it’s rhetorically very odd that he would
begin the propositional section of Part One by theorizing about single-
attribute substances without being more explicit about this in the text.

In any case, this is not our take on what happens in E1p1-E1p8.
While we agree with Gueroult that Spinoza is referring in these initial
propositions to thinking substance, extended substance, and so forth,
these substances shouldn’t be regarded as solitary atoms destined to
become divine attributes upon their synthesis into God. Rather, on
our interpretation of the text, multifaceted substance is in the picture
from the very beginning and Spinoza is simply homing in on existence of
particular kinds. From our theory-internal formal perspective, we might
say that Spinoza is theorizing in E1p1-E1p8 at the “secondary sortal
level” by considering the possible unifaceted projections of multifaceted
substance. Note that on this more palatable reading, the absolutely
infinite divine substance needn’t be constructed from the one-attribute
substances starring in E1p1-E1p8 because, from the onset, these one-
attribute substances are simply projections of multifaceted substance
having existence of this or that kind. So, we can maintain with Gueroult
that Spinoza is concerned in E1p1-E1p8 with substances of one attribute
without having to assert that these substances are later unified into God.

Although our reading differs from Gueroult’s, many of the textual
considerations put forward by Gueroult (1968), Smith (2014), and others
in favor of Gueroult’s original proposal also support our own Gueroult-
like interpretation of the text. We marshal some of this evidence in
§5.2 when we get to the places in the Ethics where it comes up. In the
meantime, we want to introduce two new considerations supporting our
Gueroult-like interpretation.

The first is a textual point from Appendix I of the Short Treatise
(KV), which Melamed (2019) argues is the earliest available draft of the
Ethics. The third axiom of this appendix states,

A3: Things [De dingen] that are distinguished really either
have different attributes, like thought and extension, or are
related to different attributes, like understanding and mo-
tion, of which the one belongs to thought, the other to ex-
tension.

On a natural reading of this axiom where the two like-phrases are treated
in parallel fashion, Spinoza offers Thought and Extension as examples
of things that are distinguished really by having different attributes,
and understanding and motion as examples of things related to different
attributes.40 Later in the appendix, Spinoza argues that two distinct

40For Spinoza, understanding (or intellect) and motion are infinite modes of
Thought and Extension respectively (see Ep. 64).
substances would have to be distinguished really (rather than modally) en route to establishing that every substance is by its nature infinite (as in E1p8) and has an essence involving existence (as in E1p7). The wording of A3 strongly suggests that the substances Spinoza is concerned with throughout Appendix I of KV are Thought (or thinking substance), Extension (or extended substance), and so forth. But then, if Melamed is correct that Appendix I is an early draft of the beginning of the Ethics, we have reason to think that Spinoza is dealing with these attributes in the opening propositions of the Ethics as well.

Admittedly, this consideration isn’t decisive, resting as it does on a particular reading of A3 of Appendix I and the assumption that there is a tight connection between this appendix and the beginning of the Ethics. As such, we don’t want to put too much weight on it. There are in any case other textual considerations from the Ethics itself supporting a Gueroult-like interpretation, which we discuss in the next section. There is also a systematic methodological consideration. Even if one finds the textual evidence for our Gueroult-like reading suggestive, one might nevertheless wonder why Spinoza would occupy himself in E1p1-E1p8 proving things about extended substance, thinking substance, and other attributes rather than theorizing about the absolutely infinite divine substance in its fullness from the get-go. There is a very plausible answer to this. As we discussed in §4, each Spinozian attribute expresses the essence of substance in that attributes inhere in themselves (E1p29s) and are conceived through themselves (E1p10). But then insofar as one is interested in the properties of substance qua substance, which for Spinoza can be deduced from the essence of substance as articulated in E1d3, it suffices to consider attributes (i.e., single-faceted substances). While Spinoza’s primary goal in Part One of the Ethics is to develop his picture of the absolutely infinite divine substance, he first wants to theorize about God qua substance. For this purpose, it makes good methodological sense to start by investigating attributes before shifting to the more radical pluralistic ℵ-perspective where things have existence of multiple kinds.41

5.2 Reconstructing E1p1-E1p15

In this section, we reconstruct the first fifteen propositions of Part One together with their demonstrations. The non-geometrical parts of the text—the scholia—are discussed only to the extent that they bear on our reconstructions. For the sake of brevity, we normally do not cite the

41That said, there is a danger here in proving things about substance in a way that depends crucially on the fact that single-faceted substances are under consideration and then unduly applying these results to multi-attribute substance. As we discuss, Spinoza flirts with this danger but it doesn’t ultimately undermine the validity of his reasoning.
full text of a demonstration. We try to trace Spinoza’s argumentation as faithfully as possible, but sometimes make minor corrections as needed (while documenting these corrections). Going forward, the reader might find it helpful to have a copy of the *Ethics* at hand so as to better judge the success of our efforts.

Spinoza considers his first proposition to be evident [*patet*] from the mere definitions of substance and mode:

\[\text{P1: A substance is prior in nature [natur\(\text{a}\)] to its affections.}\]

Given that Spinoza’s definitions E1d3/E1d3* and E1d5 are stated using only the relations of inherence and conception, the notion of priority in E1p1 should presumably be interpreted ontologically in terms of \(\leq\) or conceptually in terms of \(\sqsubseteq\) (Garrett 1990; Della Rocca 2002). The Latin ‘natur\(\text{a}\)’ (‘in nature’ as opposed to ‘in cognition [cognitione]’) might be taken to suggest ontological priority (Jarrett 1978), but E1p1 can be proven from E1d3* and E1d5\(_a\) on either construal:

**Ontological version:**
\[\text{E1p1}_o. \forall \bar{x} \forall \bar{y} ((\text{Substance}(\bar{x}) \land \text{Affection}(\bar{y}, \bar{x})) \rightarrow \bar{y} < \bar{x})^{42}\]

*Proof.* Consider \(\bar{a}, \bar{b}\) in \(\circ\) where \(\text{Substance}(\bar{a})\) and \(\text{Affection}(\bar{b}, \bar{a})\).\(^{43}\) By E1d5\(_a\), \(\bar{b} \subseteq \bar{a}\) and \(\bar{a} \neq \bar{b}\). Now suppose \(\bar{a} \leq \bar{b}\). By E1d3*, \(\bar{a} = \bar{b}\), a contradiction. Therefore, \(\bar{b} < \bar{a}\).  

**Conceptual version:**
\[\text{E1p1}_c. \forall \bar{x} \forall \bar{y} ((\text{Substance}(\bar{x}) \land \text{Affection}(\bar{y}, \bar{x})) \rightarrow \bar{y} \sqsubseteq \bar{x})\]

*Proof.* Consider \(\bar{a}, \bar{b}\) as above. By E1d5\(_a\), \(\bar{b} \subseteq \bar{a}\) and \(\bar{a} \neq \bar{b}\). Now suppose \(\bar{a} \sqsubseteq \bar{b}\). By E1d3*, \(\bar{a} = \bar{b}\) and we have a contradiction as before.  

The following analogy might be helpful: water is prior to a rotation of water, in the sense both that the rotation inheres in the water but not vice versa, and that the concept of a rotation of water requires the concept of water for its formation but the reverse isn’t true.\(^{44}\)

Spinoza claims that his second proposition is also evident from the definition of substance:

\(^{42}\)We use overlined variables in translating E1p1-E1p8 because we take Spinoza to be theorizing at the secondary sortal level throughout this part of the *Ethics*. That said, at least some of these opening propositions, like E1p1, can also presumably be applied to multifaceted entities. For instance, God as the absolutely infinite divine substance is prior in nature to its infinitely-faceted modes. As we discuss, Spinoza sometimes needs to appeal to the multifaceted versions and in these cases the question arises whether he is justified in doing so on the basis of his demonstrations.

\(^{43}\)We run through the demonstrations in this section at the actual world \(\circ\). So long as the premises employed in a proof are metaphysically necessary, \(\circ\) can be regarded as a representative metaphysically possible world and the proven result is a metaphysical necessity.

\(^{44}\)While such analogies can be useful in building intuition, it is important to keep in mind that for Spinoza water is a mode, not a substance.
P2: Two substances having different attributes have nothing in common with one another \([\text{nihil inter se commune habent}]\).

Dem.: This also evident from D3. For each must be in itself and be conceived through itself, \textit{or} the concept of the one does not involve the concept of the other.

\[\forall \bar{x} \forall \bar{y}(\text{Substance}(\bar{x}) \land \text{Substance}(\bar{y}) \land \neg \text{Same-sort}(\bar{x}, \bar{y}) \rightarrow \bar{x} \perp \bar{y})\]

While E1p2 might follow immediately from E1d3/E1d3*, it isn’t clear what Spinoza even means by “having nothing in common” and indeed many of Spinoza’s readers seem to misinterpret this expression. One natural thought is that things have nothing in common when they share no properties (as Oldenburg seems to think in Ep. 3), but this can’t be right because any two substances will share the property of inhering in oneself, the property of being conceived through oneself, the property of being self-caused (E1p7d), and so forth. Moreover, given that Spinoza cites only his definition of substance in the demonstration of E1p2, he presumably has something involving \(\leq\) or \(\sqsubseteq\) in mind. This still leaves a number of prima facie plausible translations. Here are six of them:

\textbf{HNIC}1. \(x \perp y \equiv x \not\leq y \land y \not\leq x\)
Neither thing inhere in the other.\(^{46}\)

\textbf{HNIC}2. \(x \perp y \equiv x \not\leq y \land y \not\lessdot x\)
Neither thing is conceived through the other.

\textbf{HNIC}3. \(x \perp y \equiv x \not\leq y \land y \not\leq x \land x \not\leq y \land y \not\leq x\)
A combination of HNIC\textsubscript{1} and HNIC\textsubscript{2}.

\textbf{HNIC}4. \(x \perp y \equiv \neg \exists z(x \leq z \land y \leq z))\)
The things don’t inhere in a common thing.

\textbf{HNIC}5. \(x \perp y \equiv \neg \exists z(x \leq z \land y \leq z)\)
Their concepts aren’t formed using a common concept.

\textbf{HNIC}6. \(x \perp y \equiv \neg \exists z((x \leq z \land y \leq z) \lor (x \leq z \land y \leq z))\)
A combination of HNIC\textsubscript{4} and HNIC\textsubscript{5}.

Of these options, we regard those involving both \(\leq\) and \(\lessdot\), HNIC\textsubscript{3} and HNIC\textsubscript{6}, as the top contenders given that Spinoza explicitly mentions both inherence and conception in his demonstration of E1p2. But in any case, we needn’t decide between HNIC\textsubscript{1}-HNIC\textsubscript{6} because the choice between these options doesn’t matter for the demonstrations in which

\(^{45}\)Similarly, Tschirnhaus suggests that things which differ both in essence and existence “have nothing in common” (Ep. 63 (IV/275/12)). Spinoza rejects this understanding as well (Ep. 64 (IV/278/15)).

\(^{46}\)Arguably, having nothing in common also requires that the things be of different sorts. After all, two things with the same attribute would have at least \textit{something} in common, \textit{viz.} their shared attribute. One could add the conjunct \(\neg \text{Same-sort}(x, y)\) to each of our candidate translations, thereby generating six additional options.
E1p2 is later used (E1p6d, E1p11d, E1p12d). Moreover, if \( \bar{a} \) and \( \bar{b} \) are distinct things of different sorts each of which inheres in only itself and is conceived through only itself—that is, these two things exhibit the independence of substance, in the strong sense of E1d3*—then we can prove that \( \bar{a} \perp \bar{b} \) on any of the available readings HNIC\textsubscript{1}-HNIC\textsubscript{6}.

\begin{proof}
Consider \( \bar{a}, \bar{b} \) in @ such that \textbf{Substance}(\( \bar{a} \)), \textbf{Substance}(\( \bar{b} \)), and \( \neg \textbf{Same-sort}(\bar{a}, \bar{b}) \), so \( \bar{a} \neq \bar{b} \). By E1d3*, \( \bar{a} \not\sqsubseteq \bar{b}, \bar{b} \not\sqsubseteq \bar{a} \), and \( \bar{a} \not\sqsupset \bar{b} \). So \( \bar{a} \perp \bar{b} \) when understood along the lines of HNIC\textsubscript{1}-HNIC\textsubscript{3}. Now, suppose there is some \( \bar{c} \) such that \( \bar{a} \leq \bar{c} \) and \( \bar{b} \leq \bar{c} \). By E1d3*, \( \bar{a} = \bar{b} = \bar{c} \), which contradicts \( \bar{a} \neq \bar{b} \). Likewise, there cannot be some \( \bar{c} \) such that \( \bar{a} \sqsubset \bar{c} \) and \( \bar{b} \sqsubset \bar{c} \). So \( \bar{a} \perp \bar{b} \) when understood along the lines of HNIC\textsubscript{4}-HNIC\textsubscript{6}. \hfill \blacksquare
\end{proof}

Spinoza next establishes that causes and effects must have something in common:

P3: If things have nothing in common with one another, one of them cannot be the cause of the other.

\textbf{E1p3}. \( \forall x \forall y (\bar{x} \perp \bar{y} \rightarrow (\bar{x} \not\leftrightarrow \bar{y} \land \bar{y} \not\leftrightarrow \bar{x})) \)

This is easily demonstrated using E1a4 and E1a5:

\begin{proof}
Consider \( \bar{a}, \bar{b} \) in @ where \( \bar{a} \perp \bar{b} \). By E1a5, \( \bar{a} \not\sqsubset \bar{b} \) and \( \bar{b} \not\sqsubset \bar{a} \). By E1a4, \( \bar{b} \not\leftrightarrow \bar{a} \) and \( \bar{a} \not\leftrightarrow \bar{b} \). \hfill \blacksquare
\end{proof}

Putting E1p2 and E1p3 together, Spinoza has already established that a substance of one attribute cannot be produced by a substance of a different attribute.

Spinoza ultimately wants to establish the more general claim that no substance can be produced by any other substance (E1p6). To establish this full causal independence of substance on the basis of E1p2 and E1p3, he proves that there cannot be two substances of the same attribute (E1p5). To do this, he first proves the following proposition concerning what distinguishes distinct things:

P4: Two or more distinct things [\textit{res distinctae}] are distinguished from one another, either by a difference in the attributes of the substances or by a difference in their affections.

For Spinoza, distinct unifaceted things must be either (i) different sorts of substances (extended vs. thinking substance), (ii) modes of different sorts of substance (intellect vs. motion), (iii) different modes of a single sort of substance (such as distinct bodies or ideas), (iv) a substance and mode of a particular sort (such as extended substance and a particular

\footnote{We can also prove that \( \bar{a} \perp \bar{b} \) on the six additional readings obtained by adding the extra conjunct \( \neg \textbf{Same-sort}(x, y) \).}
body), or (v) a substance of a particular sort and a mode of a substance of a different sort (such as thinking substance and a particular body). In E1p5 Spinoza wants to exclude the sixth option that there can be distinct substances of the same sort, but he first needs to establish E1p4 to put him in position to argue that distinct substances of the same sort wouldn’t be distinguishable by either attributes or modes.

In cases (i), (ii), (iii), and (v), E1p4 clearly applies. In case (i), the distinct substances are conceived as different by virtue of being different attributes. In case (ii), the distinct modes are conceived as different by virtue of being modes of different attributes. In case (iii), the distinct modes are distinguished by a difference in the affections of substance, in that the things are just different affections or modes. In case (v), the distinct things are distinguished by a difference in attributes, in that one is a substance of one attribute while the other is a mode belonging to a different attribute. However, (iv) poses a problem for Spinoza’s statement of E1p4 because it isn’t clear how a substance and a mode of this substance can be distinguished by a difference either in attributes or in affections. To accommodate this case as well, E1p4 should presumably read that two or more distinct things are distinguished from one another either by a difference in the attributes of the substances, by a difference in their affections, or by the difference between attribute and affection. Working with this restatement, we translate E1p4 as follows using the four-place relation $D(t, t', t'', t''')$ symbolizing $t$ and $t'$ are distinguished by the difference between $t''$ and $t'''$:

\[
\begin{align*}
E1p4. \forall \bar{x}\forall \bar{y} & (\bar{x} \neq \bar{y}) \rightarrow \exists \exists \exists \exists \exists D(\bar{x}, \bar{y}, \bar{z}, \bar{u}) \land \\
& ((\text{Attribute}(\bar{z}) \land \text{Attribute}(\bar{u}) \land \bar{z} \neq \bar{u}) \lor \\
& (\text{Mode}(\bar{z}) \land \text{Mode}(\bar{u}) \land \bar{z} \neq \bar{u}) \lor \\
& (\text{Attribute}(\bar{z}) \land \text{Mode}(\bar{u})) \lor \text{Mode}(\bar{z}) \land \text{Attribute}(\bar{u}))))
\end{align*}
\]

Proving E1p4 requires two tacit premises. The first allows us to infer from the fact that a thing inheres in itself that it is also conceived through itself, and is therefore a substance:

(PI) $\forall x\forall y (x \leq y \rightarrow x \sqsubseteq y)$

In words: inherence implies conceived through.\(^{49}\)

The second tacit premise is recognized by Garrett (1990), who regards it as a version of the Identity of Indiscernibles:

---

\(^{48}\)Cases (i) and (ii) correspond to ways in which things can be really distinguished according to A3 of Appendix I of KV (discussed in §5.1). Presumably, Spinoza would also count (v) as a real distinction and the remaining cases (iii) and (iv) as modal distinctions (per A2 of Appendix I of KV: “Things that are different are distinguished either really or modally”). See also Descartes’s Principles of Philosophy, I. 60-1, for closely related discussion of real versus modal distinctions.

\(^{49}\)Della Rocca (2008) suggests that E1d3 and E1d5 assume the biconditional: $a$ inheres in $b$ iff $a$ is conceived through $b$. For our reconstruction of E1p4d, only the left-to-right direction—which is much less controversial—is required.
(PII) $\forall x\forall y (x \neq y \rightarrow \exists z \exists u (D(x, y, z, u) \land z \neq u))$

In words: distinct things are distinguished by some difference in what there is.

We can now reconstruct Spinoza’s reasoning as follows:

Proof. Consider $\bar{a}$ in $\@$. By E1a1, either $\bar{a} \leq \bar{a}$ or $\bar{a} \leq \bar{b}$ for some $\bar{b} \neq \bar{a}$. If $\bar{a} \leq \bar{a}$, then $\text{Substance}(\bar{a})$ by E1d3 and PI. If $\bar{a} \leq \bar{b}$ for some $\bar{b} \neq \bar{a}$, then $\text{Mode}(\bar{a})$ by E1d5$_b$ and PI. Thus, $\text{Substance}(\bar{a})$ or $\text{Mode}(\bar{a})$ by constructive dilemma, and because $\bar{a}$ was arbitrarily chosen everything is either a substance or a mode.$^{50}$ Next, consider $\bar{a}, \bar{b}$ in $\@$ where $\bar{a} \neq \bar{b}$. By PII, there are $\bar{c}, \bar{d}$ such that $D(\bar{a}, \bar{b}, \bar{c}, \bar{d})$ and $\bar{c} \neq \bar{d}$. Given what was just established, either $\text{Substance}(\bar{c})$ and $\text{Substance}(\bar{d})$, and so $\text{Attribute}(\bar{c})$ and $\text{Attribute}(\bar{d})$ given the Projection Constraint, or $\text{Mode}(\bar{c})$ and $\text{Mode}(\bar{d})$, or $\bar{c}$ and $\bar{d}$ are of different ontological kinds. ■

With E1p4 in place, Spinoza then proves in E1p5 that there is at most one thinking substance, one extended substance, and so forth. Because it is widely believed that Spinoza’s proof of E1p5 fails, or at least requires some nonobvious fix, we quote the demonstration in full:

P5: In nature there cannot be [In rerum natura non possunt dare] two or more substances of the same nature or attribute.

Dem.: If there were two or more distinct substances, they would have to be distinguished from one another either by a difference in their attributes, or by a difference in their affections (by P4). If only by a difference in their attributes, then it will be conceded [concedetur] that there is only one of the same attribute. But if by a difference in their affections, then since a substance is prior in nature [sit prior naturâ] to its affections (by P1), if the affections are put to one side and [the substance] is considered in itself [in se considerata], i.e., (by D3 and A6), considered truly, one cannot be conceived to be distinguished from another, i.e. (by P4), there cannot be many, but only one [of the same nature or attribute], q.e.d.

We translate E1p5 as follows:

E1p5. $\neg \exists \bar{x} \exists \bar{y} (\text{Substance}(\bar{x}) \land \text{Substance}(\bar{y}) \land \bar{x} \neq \bar{y} \land \text{Same-sort}(\bar{x}, \bar{y}))$\(^{51}\)

$^{50}$ As Spinoza puts it, “Outside the intellect there is nothing except substances and their affections”. E1d5$_b$ ensures that every mode is an affection of substance.

$^{51}$ The fact that E1p5 begins “In nature...” indicates that this proposition concerns existing substances and should therefore be translated using actualist quantifiers. This actualist reading is further supported by E1p8d and E1p14d where Spinoza derives contradictions using E1p5 within redactio contexts in which there are multiple co-existing substances in play.
From E1p4, we know that two substances must be distinguished by a difference in either attributes or modes. Spinoza argues in E1p5d that neither kind of difference can distinguish two or more substances of the same nature or attribute from one another. Therefore, there cannot be distinct same-natured substances.

Both forks of Spinoza’s demonstration have been contested. The most famous objection is due originally to Leibniz in his 1678 notes on Spinoza’s *Ethics* (translated in Leibniz 1969, pp. 198–199), where he argues that Spinoza’s inference from the supposition that two substances are distinguished only by a difference in their attributes to the conclusion that “there is only one of the same attribute” isn’t valid. Can there not be substances that share some attributes but are distinguished by their failure to share others? Consider a substance $S_1$ with attributes $A_1$ and $A_2$, and another substance $S_2$ with attributes $A_1$ and $A_3$. These substances differ in their attributes yet we still have two substances of the same attribute $A_1$ (see also Boole 1854, p. 215; Bennett 1984, pp. 66–69; Garrett 1990; and Della Rocca 2002, who calls this the “different attributes problem”).

On the mode side of things, many commentators have also wondered why the priority of substance over mode in E1p1 entitles Spinoza to put affections to one side when trying to distinguish different substances (see for instance Hooker 1980; Charlton 1981; Bennett 1984, pp. 67–69; Garrett 1990; and Della Rocca 2002, who calls this the “different modes problem”). Can we not conceive two substances as distinct by appealing to a difference in their affections?

These objections have been argued to threaten the very foundation of Spinoza’s substance monism given the use of E1p5 in E1p14d, and so philosophers have gone to great lengths on Spinoza’s behalf to overcome them. In an influential paper on E1p5, Garrett (1990) considers no fewer than six possible responses to the Leibniz worry (including his own preferred interpretation), and three responses to the latter problem of different modes (including his own). However, on our interpretation the threats are overblown and we needn’t engage in any extended exegetical gymnastics to tighten up E1p5d in such a way that Spinoza’s proof goes through.

Starting with the different-modes problem, it doesn’t seem all that mysterious that the ontological and conceptual priority of substance over mode precludes us from appealing to modes in distinguishing substance. It might be helpful to consider a pedestrian analogy. Suppose I have two cups of liquid and I stir one in a clockwise rotation, the other counterclockwise. It would be rather odd for someone to claim that the liquids must be different by virtue of the fact that one is moving in a clockwise rotation while the other is moving counterclockwise. Why? The Spinozian answer: because there is an asymmetric dependence between the liquid in each cup and its rotation. On one hand, the rotation of
the liquid in each cup inheres in the liquid, but the liquid doesn’t inhere in the rotation. On the other, the concept of a rotation of, say, water is formed using the concept of water, but the concept of water doesn’t require the concept of rotation for its formation. In both an ontological and conceptual sense, to be a rotation of water is to be water but the reverse isn’t true.

This is presumably how Spinoza is thinking about things in the latter half of E1p5d. We propose that he would regard the following principles needed to fill in his demonstration as evident, or at least as evident as some of his axioms:

\[(\text{PI}) \forall \bar{x} \forall \bar{y} \forall \bar{z} \forall \bar{u} ((\text{Substance}(\bar{x}) \land \text{Substance}(\bar{y}) \land D(\bar{x}, \bar{y}, \bar{z}, \bar{u})) \rightarrow ((\text{Mode}(\bar{z}) \rightarrow \text{Affection}(\bar{z}, \bar{x})) \land (\text{Mode}(\bar{u}) \rightarrow \text{Affection}(\bar{u}, \bar{y})))) \]

In words: If two substances are distinguished by a mode, then this mode is an affection of the corresponding substance.

\[(\text{PII}) \forall \bar{x} \forall \bar{y} \forall \bar{z} \forall \bar{u} (D(\bar{x}, \bar{y}, \bar{z}, \bar{u}) \rightarrow (\bar{z} \not< \bar{x} \land \bar{z} \not< \bar{y} \land \bar{z} \not= \bar{z} \land \bar{z} \not= \bar{y} \land \bar{u} \not< \bar{x} \land \bar{u} \not< \bar{y} \land \bar{u} \not= \bar{u} \land \bar{u} \not= \bar{y})) \]

In words: Things cannot be distinguished by what they have either ontological or conceptual priority over.

In fact, Spinoza presents the following axiom (i.e, general evident truth) in Appendix I to KV:

\[A7: \text{That by which things are distinguished is by its nature [wegens syn natuur] prior to such things.}\]

Spinoza is uncharacteristically sloppy here. Technically, his axiom A7 is false because two substances of different attributes—such as extended substance and thinking substance—can be distinguished by a difference in their attributes, yet the attributes aren’t prior to the substances (on our interpretation, the attributes simply are the substances). What Spinoza should have said is our premise PII: That by which things are distinguished is not posterior to such things.

Turning to the different-attributes problem, we also do not find this objection as troubling as it is often taken to be. Note that “Leibniz situations” (to borrow Smith’s 2014 term) where two substances share some but not all attributes arise only if we are dealing with substances of more than one attribute. However, on our Gueroult-like interpretation of the text, Spinoza is considering only single-faceted substances at this point in the Ethics. The problematic Leibniz situations are off the radar, and E1p5 is not meant to rule them out.\(^{52}\)

\(^{52}\)While Garrett (1990) concedes that the Leibniz worry arises only if we are dealing with substances of more than one attribute, he thinks E1p5 “obviously” concerns multi-attribute substances given its use in E1p14d. One might also point
Indeed, like Smith (2014), we take the very fact that Spinoza doesn’t say anything to address the Leibniz worry—which is fairly obvious if one is considering multifaceted substances—as evidence that he is theorizing at the secondary sortal level rather than the pluralistic $\aleph$-level. While there have been some ingenious efforts to show that Leibniz situations are incompatible with Spinoza’s broader philosophy (see Garrett 1990; Della Rocca 2002), part of what needs to be explained here is why Spinoza simply ignores the possibility of Leibniz situations in E1p5d. It’s difficult to make sense of the brevity of the first part of this proof unless Spinoza is reasoning about substances of one attribute, which clearly cannot be distinguished by a difference in attributes unless they are substances of different attributes.

To fill in the remaining gap in Spinoza’s demonstration, we need this third tacit premise:

\[(\text{PIII}) \forall \bar{x} \forall \bar{y} \forall \bar{z} \forall \bar{u} ((\text{D}(\bar{x}, \bar{y}, \bar{z}, \bar{u}) \land \text{Attribute}(\bar{z}) \land \text{Attribute}(\bar{u}) \land \bar{z} \neq \bar{u}) \rightarrow \neg \text{Same-sort}(\bar{x}, \bar{y}))\]

In words: Two single-faceted things distinguished by a difference in attributes are of different sorts.

This premise seems unobjectionable: two distinct things clearly cannot be conceived as different on the basis of their attributes if these things are both extended, both thinking, and so forth. While the relevant instances of PIII for the purpose of E1p5d are those in which we are distinguishing substances with different attributes, this premise applies equally well to modes—two modes distinguished by a difference in the attributes, such as understanding (an infinite mode of Thought) and motion (an infinite mode of Extension), must also be of different sorts.

With PI-PIII to wield, we can finally complete E1p5d as follows:

\[\text{Proof. Consider } \bar{a}, \bar{b} \text{ in } \mathbb{A} \text{ such that } \text{Substance}(\bar{a}), \text{Substance}(\bar{b}), \text{ and } \bar{a} \neq \bar{b}. \text{ By E1p4, there exist } \bar{c}, \bar{d} \text{ such that } \text{D}(\bar{a}, \bar{b}, \bar{c}, \bar{d}) \text{ where } \bar{c} \text{ and } \bar{d} \text{ are distinct attributes, are distinct modes, or belong to different ontological categories (one is a substance, the other a mode). Suppose that } \text{Attribute}(\bar{c}), \text{Attribute}(\bar{d}), \text{ and } \bar{c} \neq \bar{d}. \text{ By PIII, } \neg \text{Same-sort}(\bar{a}, \bar{b}): \text{ “It will be conceded that there is only one [substance] of the same attribute”}. \text{ Suppose next that } \text{Mode}(\bar{c}), \text{Mode}(\bar{d}), \text{ and } \bar{c} \neq \bar{d}. \text{ By PI, Affection}(\bar{c}, \bar{a}) \text{ and Affection}(\bar{d}, \bar{b}), \text{ and so } \bar{c} < \bar{a} \text{ and } \bar{d} < \bar{b} \text{ by E1p1o, and } \bar{c} \sqsubseteq \bar{a} \text{ and } \bar{d} \sqsubseteq \bar{b} \text{ by E1p1c}. \text{ But then } \neg \text{D}(\bar{a}, \bar{b}, \bar{c}, \bar{d}) \text{ by PII—a contradiction. In fact, if either } \text{Mode}(\bar{c}) \text{ or } \text{Mode}(\bar{d}), \text{ we have a contradiction by this same reasoning. Thus, it can only be that } \text{Attribute}(\bar{c}) \text{ and } \text{Attribute}(\bar{d}) \text{ (by E1p4), and so there cannot be } \bar{a}, \bar{b} \text{ such that } \text{Substance}(\bar{a}), \text{Substance}(\bar{b}), \text{ to the use of E1p5 in E1p13d where Spinoza proves the indivisibility of absolutely infinite substance. However, our treatment of E1p14d and E1p15d will require only that E1p5 applies to attributes (in this we agree with Smith 2014).} \]

\[\vec{a} \neq \vec{b}, \text{ and } \text{Same-sort}(\vec{a}, \vec{b}): \text{“There cannot be many [substances], but only one [of the same nature or attribute]”}.\]

Why does Spinoza say in E1d5d that when affections are put to one side and substance is considered in itself it is “(by D3 and A6), considered truly”? Given that E1d3 expresses the essence of substance as that which is in itself and conceived through itself, to set affections aside is to remove what is inessential to substance from consideration. Spinoza’s reference to E1a6 (“A true idea must agree with its object”) suggests that this is precisely what is involved in having a true idea of substance—such an idea “agrees” with its object (the substance) in the sense that it conceives of this substance in terms of its essential features—as being in itself and conceived through itself (rather than being in or conceived through something else)—and leaves the inessential features of substance (its affections) out of the picture.

Spinoza is now in a position to establish that substances are causally independent of each other:

**P6:** One substance cannot be produced by another substance.

**E1p6.** \(\neg \exists \vec{x} \exists \vec{y} (\text{Substance}(\vec{x}) \land \text{Substance}(\vec{y}) \land \vec{x} \neq \vec{y} \land \vec{x} \leadsto \vec{y})\)

The demonstration of E1p6 is straightforward and requires no hidden premises:

*Proof.* Consider \(\vec{a}, \vec{b}\) in @ where \(\text{Substance}(\vec{a}), \text{Substance}(\vec{b})\), and \(\vec{a} \neq \vec{b}\). By E1p5, \(\neg \text{Same-sort}(\vec{a}, \vec{b})\). By E1p2, \(\vec{a} \perp \vec{b}\). By E1p3, \(\vec{a} \not\rightarrow \vec{b}\) and \(\vec{b} \not\rightarrow \vec{a}\). ■

Extending this result, Spinoza also proves that a substance cannot be caused by any other thing. Spinoza calls this a “corollary [corollarium]” of E1p6 because he presumably takes it to be obvious that a substance cannot be caused by a mode (so obvious in fact that he doesn’t even bother to prove this piece of the argument, though we make it explicit in our reconstruction of E1p6cd):

**Cor.:** From this it follows that a substance cannot be produced by anything else.

**E1p6c.** \(\forall \vec{x} (\text{Substance}(\vec{x}) \rightarrow \neg \exists \vec{y} (\vec{x} \neq \vec{y} \land \vec{y} \leadsto \vec{x}))\)

*Proof.* Consider \(\vec{a}, \vec{b}\) in @ where \(\text{Substance}(\vec{a})\) and \(\vec{a} \neq \vec{b}\). From E1p4d, everything is either a substance or a mode, so \(\text{Substance}(\vec{b})\) or \(\text{Mode}(\vec{b})\). If \(\text{Substance}(\vec{b})\), then \(\vec{b} \not\rightarrow \vec{a}\) by E1p6. If \(\text{Mode}(\vec{b})\), then \(\vec{a} \not\subseteq \vec{b}\) by E1d3* and E1d5b, and so \(\vec{b} \not\rightarrow \vec{a}\) by E1a4. ■

Before turning to E1p7, Spinoza offers this alternative demonstration of E1p6c (this is the first but definitely not the only place in the *Ethics* where Spinoza offers multiple proofs of a proposition):
Proof. Consider $\bar{a}, \bar{b}$ in @ where $\text{Substance}(\bar{a})$, $\bar{a} \neq \bar{b}$, and $\bar{b} \leadsto \bar{a}$. By E1a4, $\bar{a} \sqsubseteq \bar{b}$, so $\neg \text{Substance}(\bar{a})$ by E1d3*—a contradiction. ■

The existence of this second route to E1p6c is important. Unlike the first proof of E1p6c, which depends on E1p5 and therefore on the fact that Spinoza is considering one-attribute substances at this point in the Ethics (otherwise E1p5d faces the Leibniz worry, which Spinoza doesn’t even think to address), the alternative proof of E1p6c depends on only the definition of substance (E1d3*) together with the causal axiom (E1a4). Because E1d3* and E1a4 pertain to both one-attribute and many-attribute substances, this second demonstration shows that the causal independence of substance asserted in E1p6c needn’t depend on whether Spinoza is considering substances of one or many attributes and can be safely imported into the pluralistic $\aleph$-setting (cf. Gueroult 1968, p. 122).

Spinoza can now prove the fundamental result that substance is a causa sui:

**E1p7.** It pertains to the nature of a substance to exist.

$\forall \bar{x} (\text{Substance}(\bar{x}) \rightarrow \Box \exists \bar{x} \text{Exists}(\bar{x}))$

To prove E1p7, we need the following equivalence mentioned in note 31:

$$(\text{PI}) \forall x (\text{Causa-sui}(x) \equiv x \leadsto x)$$

Spinoza’s proof also requires the following version of the Principle of Sufficient Reason (PSR), which he states explicitly later in E1p8s2 (see also E1p11d2, TdIE §92):

There must be, for each existing thing, a certain cause on account of which it exists.

$$(\text{PII}) \forall x \exists y (y \leadsto x)^{54}$$

More generally, Spinoza is committed to a strong version of the PSR which requires sufficient reason not only for the existence of things but also for their non-existence (E1p11d2; see Melamed & Lin 2018 for a taxonomy of PSR-variants in Spinoza’s writings, and LeBuffe 2018 for an important attempt to situate the PSR within the broader context of Spinoza’s discussion of reason [ratio]).^{55}

The demonstration of E1p7 is trivial:

^{53}While it might be thought that the second proof of E1p6c renders much of what was proven before redundant, this isn’t the case—Spinoza still uses E1p2 and E1p5 in later demonstrations of the Ethics (such as E1p8d, E1p11d2, E1p12d, E1p13d, and E1p14d).

^{54}That this version of the PSR for existing things is needed in the proof of E1p7 supports an actualist reading of this proposition. This will be particularly important in the context of E1p11d1.

^{55}E1a2 might also be read as a statement of the PSR insofar as it asserts that everything must be conceivable (or explainable; see Della Rocca 2008).
Proof. Consider $\bar{a}$ in $\mathcal{@}$ where $\text{Substance}(\bar{a})$. By the version of the PSR in PII applied at the secondary sortal level, there is some $\bar{b}$ such that $\bar{b} \sim \bar{a}$. By E1p6c, $\bar{a} = \bar{b}$, so $\bar{a} \sim \bar{a}$. By PI, $\text{Causa-sui}(\bar{a})$, so $\square_a \text{Exists}(\bar{a})$ by E1d1$_a$.

Note that given the alternative demonstration of E1p6c, E1p7 can be proven using only E1d1$_a$, E1d3*, E1a4, and our two tacit premises PI and PII, none of which depends for its validity on whether it applies to one-attribute or multi-attribute substances (this will be important later on when we get to E1p11d1).

Spinoza rounds out the first ‘stanza’ of the Ethics by establishing another fundamental property of substance:

P8: Every substance is necessarily infinite.

**E1p8.** $\forall \bar{x}(\text{Substance}(\bar{x}) \rightarrow \square \text{Inf-in-kind}(\bar{x}))$

Unlike E1p7 which can safely be applied to substances of one and many attributes, E1p8 is meant to apply only to one-attribute substances—the relevant notion of infinity here is *infinite in its own kind* (from E1d2 and E1d6e) rather than the *absolute infinity* of divine substance (from E1d6). To see this, we quote Spinoza’s demonstration in full:

Dem.: A substance of one attribute does not exist unless it is unique (P5) and it pertains to its nature to exist (P7). Of its nature, therefore, it will exist either as finite or as infinite. But not as finite. For then (by D2) it would have to be limited by something else of the same nature, which would also have to exist necessarily (by P7), and so there would be two substances of the same attribute, which is absurd (by P5). Therefore, it exists as infinite, q.e.d.

Note that this demonstration begins “A substance of one attribute...”, which, as noted by Gueroult (1968), pp. 126–127, and others, supports the claim that Spinoza is considering one-attribute substances at this point in the Ethics (and in previous propositions, given the use of E1p5 in E1p8d). Spinoza also appeals to E1d2 (the definition of *finite in its kind*) in the subproof of the argument where he derives a contradiction from the supposition that an arbitrary substance is finite. So, on the natural interpretation of E1p8d, Spinoza proves that any single-faceted substance (i.e., attribute) must exist as infinite in its own kind:

Proof. Consider $\bar{a}$ in $\mathcal{@}$ where $\text{Substance}(\bar{a})$ ($\bar{a}$ is the unique substance of its sort and $\square_a \text{Exists}(\bar{a})$ by E1p5 and E1p7 respectively). Either $\text{Finite-in-kind}(\bar{a})$ or $\text{Inf-in-kind}(\bar{a})$. If $\text{Finite-in-kind}(\bar{a})$, then $\Diamond \exists \bar{y}(\bar{a} \neq \bar{y} \land \bar{a} \not\sim \bar{y} \land \text{Same-sort}(\bar{a}, \bar{y}) \land \text{Substance}(\bar{y}))$ by E1d2, so there is some meta-physically possible world $w \in \mathcal{W}$ and $\bar{b}$ in $w$ such that $\bar{a} \neq \bar{b}$, $\bar{a} \not\sim \bar{b}$, $\text{Same-sort}(\bar{a}, \bar{b})$, and $\text{Substance}(\bar{b})$. By E1p7, $\square_b \text{Exists}(\bar{b})$. But then both
$\bar{a}$ and $\bar{b}$ exist in $\mathbb{W}$—contradicting E1p5. In fact, if $\text{Finite-in-kind}(\bar{a})$ at any metaphysically possible world in $\mathbb{W}$, this contradicts E1p5 by similar reasoning, so $\square \text{Inf-in-kind}(\bar{a})$.

Spinoza sketches an alternative demonstration of E1p8 in E1p8s1, but we omit discussion of this demonstration for the sake of space and because reconstructing it seems to require a more fine-grained model theory that distinguishes between different ‘regions’ of extension, thought, and other kinds of existence within a world.

By the time he reaches E1p9, Spinoza has shifted to the pluralistic $\aleph$-perspective from which he gradually unfolds his argument against the commonly accepted Cartesian view that each substance has only one principal attribute that constitutes its essence. At this point, things with multiple attributes are explicitly in view:

P9: The more reality or being [$\text{realitatis, aut esse}$] each thing has, the more attributes belong to it.

Spinoza considers reality and being [$\text{esse}$] as coming in degrees. He elaborates on this graded conception of reality in E1p10s:

So it is far from absurd to attribute many attributes to one substance. Indeed, nothing in nature is clearer than that each being [$\text{ens}$] must be conceived under some attribute, and the more reality, or being [$\text{esse}$] it has, the more it has attributes which express necessity, or eternity, and infinity. And consequently there is also nothing clearer than that a being absolutely infinite must be defined (as we taught in D6) as a being that consists of infinite attributes, each of which expresses a certain eternal and infinite essence.

We find similar claims in the early Short Treatise, where Spinoza offers the following gloss on his characterization of God as “a being of which all, or infinite, attributes are predicated”:

The reason for this is that since Nothing [$\text{de Niet}$] can have no attributes, the All [$\text{de Al}$] must have all attributes; and just as Nothing has no attributes because it is nothing, Something [$\text{de Iet}$] has attributes because it is something. So the more it is Something, the more attributes it must have. Consequently, God, being most perfect, infinite, and the Something-that-is-all [$\text{de alle Iet zynde}$], must also have infinite, perfect, and all attributes. (KV I 2 (I/19/10-15)).

At one extreme, there is nothingness, which has no attributes. This is an endpoint of a spectrum along which things can have more and more reality or being (i.e., somethingness), which is correlated with having
more attributes. The other endpoint is God, the “Something-that-is-all”, which has all the attributes.

Gueroult (1968), pp. 145–146, traces Spinoza’s argument at E1p9 to Descartes and his contemporaries, who accepted the principle that “nothingness cannot have any properties” (see for example Descartes’s July 29th letter to Arnauld (CSM III 358—AT V 223)). However, E1p9 and the above passages from E1p10s and the Short Treatise are formulated in terms of ‘attributes’ rather than ‘properties’, and this is a significant difference given that only Spinozian attributes and not Cartesian (or Spinozian) properties involve existence. In this paper, we have adopted Garrett’s (2017) interpretation of Spinoza as a proponent of a strong ontological pluralism according to which Thought, Extension, and other attributes are kinds of existence. Spinoza’s association of degrees of reality with the number of attributes belonging to a thing fits comfortably within this interpretation. As Garrett puts it,

“If each attribute constitutes a different manner of existence, then it is natural to infer, as Spinoza does, that greater total reality is correlated with a greater number of attributes. (p. 28)

Formally, we translate E1p9 (which also appears in Ep. 9 (IV/45/20-21)) using the has at least as much reality or being relation $\leq_R$ and has at least as many attributes relation $\leq_A$:

$\textbf{E1p9. } \forall x_N \forall y_N (x_N \leq_R y_N \rightarrow x_N \leq_A y_N)$

Spinoza takes E1p9 to be evident from E1d4, though (as usual) it isn’t entirely clear how the demonstration is supposed to work. Given that Spinoza previously established in E1p7d that the essence of substance involves existence, and given the definition of attribute (E1d4) as what constitutes the essence of substance, one option is to interpret ‘$\leq_R$’ as expressing that one thing has at least as much reality as another iff the first thing has at least as many distinct projections involving existence (i.e., projections that are substance per E1p7):

$\mathcal{M}, g, w \models t_R \leq_R t'_R \text{ iff } |\{ \bar{x} : \exists s(\pi_s(w)([t_R]_{\mathcal{M}, g, w}) = \bar{x} \wedge \text{Substance}(\bar{x}))\}| \leq |\{ \bar{x} : \exists s(\pi_s(w)([t'_R]_{\mathcal{M}, g, w}) = \bar{x} \wedge \text{Substance}(\bar{x}))\}|$

If we then interpret ‘$\leq_A$’ as expressing that a thing has at least as many attributes belonging to it as another iff the first thing has at least as many distinct projections that are attributes as the second thing,

$\mathcal{M}, g, w \models t_R \leq_A t'_R \text{ iff } |\{ \bar{x} : \exists s(\pi_s(w)([t_R]_{\mathcal{M}, g, w}) = \bar{x} \wedge \text{Attribute}(\bar{x}))\}| \leq |\{ \bar{x} : \exists s(\pi_s(w)([t'_R]_{\mathcal{M}, g, w}) = \bar{x} \wedge \text{Attribute}(\bar{x}))\}|$

then E1p9 follows trivially from E1d4:

\footnote{In his Principles of Philosophy, I. 52, Descartes refers to this principle as a “common notion”—i.e., an axiom.}
Proof. Consider $a_R$, $b_R$ in $@$ where $a_R \leq_R b_R$. By E1d4 and the above interpretations of $\leq_R$ and $\leq_A$, $a_R \leq_A b_R$.\footnote{For a different but related take on E1p9d, see Brandau (2015), who argues that Spinoza also considers essence as coming in degrees and that a thing’s degree of essence is correlated with both its degree of reality and the plurality of its attributes.} ■

In his next proposition E1p10, Spinoza further elucidates his strong ontological pluralist position by establishing that although things can have existence of different kinds (i.e., belong to different attributes), these strata of existence are conceptually isolated from one another:

\[
P10: \text{Each attribute of a substance must be conceived through itself.}
\]

**E1p10.** $\forall \bar{x}\left(\text{Attribute}(\bar{x}) \rightarrow \bar{x} \subseteq \bar{x}\right)$

Della Rocca (1996) calls E1p10 the “conceptual barrier” between the attributes. It follows immediately from Spinoza’s definitions of substance and attribute:

Proof. Consider $\bar{a}$ in $@$ where $\text{Attribute}(\bar{a})$. By E1d4, $\text{Substance}(\bar{a})$ and so $\bar{a} \subseteq \bar{a}$ by E1d3. ■

According to E1d4, an attribute constitutes the essence of substance, where this essence is *being in (only) itself and conceived through (only) itself* (as expressed by E1d3). Thus, insofar as it constitutes the essence of substance, an attribute must be conceived through itself (and in itself, though this does not currently interest Spinoza and is mentioned only in E1p29 (II/71/10)).

Given the conceptual barrier between the attributes, a reader might be tempted to conclude that extended substance, thinking substance, and so forth are truly distinct substances. However, Spinoza warns in a scholium following E1p10 that this would be a mistake:

But if someone now asks by what sign we shall be able to distinguish the diversity of substances, let him read the following propositions, which show that in Nature there exists only one substance, and that it is absolutely infinite. So that sign would be sought in vain.

The first of these propositions en route to Spinoza’s substance monism is the claim that God necessarily exists:

\[
P11: \text{God, or a substance consisting of infinite attributes, each of which expresses eternal and infinite essence, necessarily exists.}
\]
It’s tempting to translate E1p11 as ‘□ Exists(God)’ using the constant symbol ‘God’ to refer to God. However, from a logical point of view this would be problematic, given that Spinoza establishes the uniqueness of God only later in E1p14. Instead, we translate E1p11 using the God-predicate ‘God(x)’ defined in E1d6:

\[ \exists x (\text{God}(x) \land \Box \exists x) \]

After Spinoza proves that God exists and is unique, we can interpret the constant ‘God’ as a name for the absolutely infinite divine substance.

Spinoza offers no fewer than four demonstrations of E1p11. In the interest of space, we discuss only the first and second proofs. The first is strikingly brief:

Dem.: If you deny this, conceive, if you can, that God does not exist. Therefore (by A7) his essence does not involve existence. But this (by P7) is absurd. Therefore God necessarily exists, q.e.d.

This demonstration is prima facie problematic because Spinoza seems to apply E1p7 to absolutely infinite substance, whereas we earlier adopted a Gueroult-like interpretation of this proposition on which it pertains only to substances of one attribute. However, as we discussed, Spinoza’s alternative proof of E1p6c entitles him to invoke an unrestricted version of E1p7 applicable to both one-attribute and many-attribute substances (i.e., with non-overlined variables), which we now call E1p7*.

There is a more pressing concern. Even if we let Spinoza help himself to this unrestricted principle, one might still worry that he begs the question in E1p11d1 given that E1p7* involves actualist quantification over only the existing things in a world, telling us that any existing substance in the world of evaluation has an essence involving existence. It seems that at best Spinoza’s demonstration bootstraps God’s existence into necessary existence—if God exists, it is a substance (by E1d6) and therefore it necessarily exists (by E1p7*)—but his proof doesn’t establish that God in fact exists, which is part of what Spinoza attempts to show.

Garrett (1979) recognizes this worry and tries to rescue E1p11d1 on Spinoza’s behalf by arguing that Spinoza is entitled to the claim at E1p7

\footnote{The translation using ‘God’ might also seem problematic given how Spinoza is in the midst of proving that God even exists, but our model theory allows for constants to denote nonexistent.}

\footnote{Three of these demonstrations are explicitly labeled as such. The fourth is found in the opening paragraph of E1p11s, where it is presented as an “a priori” recasting of the third “a posteriori” proof.}

\footnote{One might likewise worry about the use of E1d6, which is also formulated using actualist quantifiers (Jarrett 1976, 1978; see also Lin 2007 for related discussion). The discussion here could have been framed in terms of E1d6.}

\footnote{For a dissenting view according to which God’s existence \textit{simpliciter} is never in question in the \textit{Ethics}, see Barcan Marcus (1986).}
that it pertains to the nature of any possible substance to exist. Garrett offers some fairly complicated reasoning to strengthen E1p7 (drawing on Spinoza’s second proof of E1p11 discussed below), but there is no indication from the brief text of E1p7d that Spinoza had anything like it in mind at this earlier point in the *Ethics*. In any case, this heavy-duty repair isn’t necessary (as Garrett 2018, p. 56, himself seems to recognize in a recent postscript to the 1979 paper). So long as the version of the PSR used in E1p7d (from E1p8s2: “There must be, for each existing thing, a certain cause on account of which it exists”) is a necessary truth, and Spinoza’s definitions, axioms, and other tacit premises needed to establish E1p7* are also necessary, we can regard E1p7* as a necessary truth holding at each metaphysically possible world in a model. For E1p11d1 to go through, all Spinoza then needs is the extra premise that God possibly exists—or more precisely, that it is possible for there to exist an absolutely infinite substance (Jarrett 1976, 1978; Wilson 1981; Della Rocca 2002 make similar observations).  

$$
(PI) \ \diamond \exists x_R \text{God}(x_R)
$$

**Proof.** God possibly exists by PI, so there is a metaphysically possible world \(w \in W\) and \(a_R\) in \(w\) such that \(\text{God}(a_R)\). Suppose \(\boxdot a_R \rightarrow \exists \text{Exists}(a_R)\). By E1a7, \(\square a_R \exists \text{Exists}(a_R)\). But \text{Substance}(a_R)\) by E1d6, so \(\square a_R \exists \text{Exists}(a_R)\) by E1p7*—a contradiction. Therefore, \(\neg \boxdot a_R \rightarrow \exists \text{Exists}(a_R)\), so \(\square a_R \exists \text{Exists}(a_R)\) by E1d1\,a and E1d1\,b, and \(\square \exists \text{Exists}(a_R)\) given that being necessitated by the essence of any thing entails metaphysical necessity.  

Crucially, E1d1\,a, E1d1\,b, E1d6, E1a7, and E1p7* are all employed at the metaphysically possible world \(w\) where \(a_R\) exists, and this needn’t be the actual world \(@\), so Spinoza’s demonstration doesn’t prejudge the issue of whether God exists.  

Spinoza’s second demonstration of E1p11 begins from the following more general two-sided version of the PSR pertaining both to existence and to nonexistence:

---

62 This premise is also common in contemporary Anselmian ontological arguments for God’s existence. See for example Hartshorne (1965, 1967) and Plantinga (1974).  
63 As Garrett (1979) recognizes, Spinoza’s reasoning in E1p11d is unnecessarily circuitous. The heart of the proof is the inference from God being a substance to the conclusion that God’s essence involves existence; the surrounding material involving conceivability isn’t really needed. In his 1979, Garrett conjectures that the inclusion of this fluff simply reflects Spinoza’s preference for *reductio* arguments and his desire to use both the essence-based and conceivability-based characterizations of a *causa sui*. In his 2018 postscript, Garrett more interestingly suggests that “proceeding in this less direct way allows Spinoza to urge his readers to try for themselves the experiment of seeking to conceive of God as not existing—an attempt that, if Spinoza is right, will lead the reader, in its failure, to experience instead the intellectual intuition of God’s existence in the ontological argument” (p. 57).
For each thing there must be assigned a cause, or reason, as much for its existence as for its nonexistence. For example, if a triangle exists, there must be a reason or cause why it exists; but if it does not exist, there must also be a reason or cause which prevents it from existing, or which takes its existence away.

To formalize this principle, the following new notation will be helpful:

\[ x \sim^+ y \equiv \exists y \ (x \sim y) \]
Read: \( x \) is a cause of the existence of \( y \).

\[ x \sim^- y \equiv \neg \exists y \ (x \sim y) \]
Read: \( x \) is a cause of the nonexistence of \( y \).

Here, a thing \( a \) is a cause of its own existence (i.e., a causa sui) iff \( a \sim^+ a \), and a cause of its own nonexistence (i.e., a Chimera) iff \( a \sim^- a \).

Because the above PSR principle, which serves as the opening premise of E1p11d2, concerns both existing and non-existing things (and even necessarily non-existing things), we need possibilist quantification with ‘\( \Pi \)’/’\( \Sigma \)’ over the global domains in a Spinozian model in order to translate it:

\[(\Pi) \ \Pi x ((\exists y (y \sim^+ x) \rightarrow \Sigma y (y \sim x))) \land (\neg \exists y (y \sim^- x) \rightarrow \Sigma y (y \sim x)))\]

In words: if a thing exists then there is a cause for its existence, and if a thing doesn’t exist then there is a cause for its nonexistence.

Spinoza’s strategy in E1p11d2 is to show that there cannot be a cause or reason for God’s nonexistence. Assuming that PI is a necessary truth, and that things either exist or fail to exist at a world, he can then infer that God necessarily exists.

Spinoza first argues that a reason for God’s nonexistence couldn’t reside outside of God:

But if there were such a reason, or cause, it would have to be either in God’s very nature or outside of it, i.e., in another substance of another nature. For if it were of the same nature, that very supposition would concede that God exists. But a substance which was of another nature would have nothing in common with God (by P2), and therefore could neither give him existence nor take it away.

Reconstructing the argumentation in this passage requires the premise that God’s nonexistence could be caused only by a substance:

\[(\Pi I) \ \Pi x (\text{God}(x) \rightarrow \Pi y (y \sim^- x \rightarrow \text{Substance}(y)))\]

[64]Strictly speaking, the implicit premises PI and PII employed in E1p7d should be reformulated using \( \sim^+ \). This wouldn’t affect the validity of Spinoza’s earlier proof.
This premise is left tacit, presumably because Spinoza takes PII to be sufficiently obvious to the reader. However, PII might be justified using reasoning similar to that found in Spinoza’s first demonstration of E1p6c, where it is established that the existence of a substance cannot be caused by a mode.\textsuperscript{65} We also need the premise that if God’s nonexistence has an external cause (i.e., a cause that does not inhere in God), then this external cause exists. While Spinoza takes Chimeras to be causes of their own nonexistence, this seems to be an exceptional case—he refers only to existing things as external causes of the existence or nonexistence of other things (see Garrett 1979, p. 208 for discussion).

\begin{equation}
\Pi x_R (\text{God}(x_R) \rightarrow \Pi y_R ((y_R \rightsquigarrow - x_R \land y_R \not\in x_R) \rightarrow \text{Exists}(y_R)))
\end{equation}

This premise, which is also left implicit, is needed for the part of the proof where Spinoza infers from the supposition that God’s nonexistence is caused by another substance of the same nature (i.e., a substance exhibiting absolute infinity) that God exists, and as a result he disregards this fork because it concedes what he is trying to prove. The last part of the argument requires the principle that God’s nonexistence couldn’t be caused by a substance of another nature (a non-God):

\begin{equation}
\Pi x_R (\text{God}(x_R) \rightarrow \neg \Sigma y_R ((y_R \rightsquigarrow - x_R \land \text{Substance}(y_R) \land \neg \text{God}(y_R)))
\end{equation}

Spinoza cites E1p2 in justifying PIV, but he could also have argued for the more general claim that God’s nonexistence couldn’t be caused by any other substance—absolutely infinite or otherwise—by running the kind of argument in his alternative demonstration of E1p6c involving (possibilized versions of) the causal axiom (E1a4) and the definition of substance (E1d3).\textsuperscript{66}

In the second fork of E1p11d2, Spinoza argues that the cause of God’s nonexistence couldn’t reside in God’s nature either:

Since, then, there can be, outside the divine nature, no reason, or, cause which takes away the divine existence, the reason will necessarily have to be in his nature itself, if indeed he does not exist. That is, his nature would involve a contradiction. But it is absurd to affirm this of a Being absolutely infinite and supremely perfect.

It would have been nice to hear more from Spinoza about why God cannot necessitate its own nonexistence—Spinoza says only that God’s nature would therefore involve a contradiction, which is “absurd”. Given

\begin{footnote}{65} One would need to appeal to possibilist versions of the definitions of substance and mode, the causal axiom, and so forth. \end{footnote}

\begin{footnote}{66} This argument would require a variant of the causal axiom formulated using \(\rightsquigarrow\): the cognition of a non-existing thing depends on the cognition of the cause of its nonexistence. \end{footnote}
this lack of elaboration, we simply treat it as a premise of the argument that an absolutely infinite substance cannot be a Chimera (see Della Rocca 2002 for an attempt to motivate this premise on Spinoza’s behalf):

\[(PV) \Pi x_R(\text{God}(x_R) \rightarrow x_R \not\sim x_R)\]

Putting things together, Spinoza concludes that God necessarily exists:

Therefore, there is no cause, or reason, either in God or outside God, which takes his existence away. And therefore, God necessarily exists, q.e.d.

**Proof.** Consider \(a_R\) where \(\text{God}(a_R)\). By the version of the PSR in PI applied at the \(R\)-perspective, if \(a_R\) exists in any metaphysically possible world \(w \in \mathcal{W}\), then there is some \(b_R\) such that \(b_R \sim^+ a_R\), and if \(a_R\) does not exist in \(w\), then there is some \(b_R\) such that \(b_R \sim^- a_R\). Suppose that \(a_R\) does not exist in \(w\), so \(b_R \sim^- a_R\) where \(\text{Substance}(b_R)\) by PII. Either \(b_R \leq a_R\) or \(b_R \not\leq a_R\). If \(b_R \not\leq a_R\) (i.e., \(b_R\) is an external cause), then \(b_R\) exists in \(w\) by PIII. If \(\text{God}(b_R)\), then E1p11 follows easily, so let us ignore this option. If \(\neg\text{God}(b_R)\), this contradicts PIV. On the other hand, if \(b_R \leq a_R\) (i.e., \(b_R\) is an internal cause) and so \(b_R = a_R\) by E1d3*, then \(a_R \sim^- a_R\)—contradicting PV. Either way, there cannot be some \(b_R\) such that \(b_R \sim^- a_R\): “There is no cause, or reason which takes [God’s] existence away”. Therefore, assuming that a thing exists or fails to exist in any world \(w \in \mathcal{W}\), \(a_R\) exists in every metaphysically possible world—that is, \(\Box\text{Exists}(a_R)\). ■

While this second demonstration is more complicated than E1p11d1, it allows us to see from a different angle why E1p11 holds by focusing on causes for God’s nonexistence rather than existence (and these different angles are reflected in the different versions of the PSR underlying the two demonstrations).

The premise that God is a possible substance from E1p11d1 and the premise that God cannot necessitate its own nonexistence from E1d11d2 are especially significant given an important outstanding challenge to Spinoza’s two proofs recognized by Garrett (1979). As Garrett observes, neither E1p11d1 nor E1p11d2 appeals to the condition that God consists of an infinity of attributes, so it seems that either of these proofs could serve equally well to establish that substances of fewer-than-all attributes necessarily exist (such as Cartesian merely extended substances, merely thinking substances, and so forth). To rein in the scope of E1p11d1 and E1p11d2 to only absolutely infinite substance, Spinoza requires that something like the following principle be a necessary truth:

**Non-God Substances are Chimeras:**
\[\Pi x_R((\text{Substance}(x_R) \land \neg\text{Abs-infinite}(x_R)) \rightarrow x_R \sim^- x_R)\]
Garrett argues that Spinoza anticipates this generalizability worry and his third and fourth proofs of E1p11 are presented in part to forestall it; unlike the first two proofs, the latter two appeal to the condition that God has infinite attributes. Della Rocca (2002) and Lin (2007) also offer arguments on Spinoza’s behalf using variants of the Principle of the Identity of Indiscernibles and PSR to show that substances besides God that are not absolutely infinite are impossible for internal reasons. In the rest of this paper, we let Spinoza help himself to the “Non-God Substances are Chimeras” principle as a necessary truth (which also seems to be required in E1p13d and E1p13cd), though we leave further discussion of its justification for another occasion.

In propositions E1p12 and E1p13 that follow, Spinoza argues that substance is indivisible:

P12: No attribute of a substance can be truly conceived from which it follows that the substance can be divided [Nullum substantiae attributum potest vere concipi, ex quo sequatur substantiam posse dividi].

P13: A substance which is absolutely infinite is indivisible.

It’s noteworthy that there are two propositions here, the first concerned with the indivisibility of single-faceted substances (i.e., attributes) and the second with the indivisibility of absolutely infinite substance. The inclusion of both propositions supports our interpretation of the start of the Ethics, according to which Spinoza is theorizing about substance at different levels. Formally speaking, E1p12 quantifies over the secondary sortal domains assigned to a world:

E1p12. ∀x(Substance(x) → ¬Divisible(x))

In contrast, E1p13 quantifies over the primary ℵ-domain (E1d6 ensures that something is absolutely infinite only if it is a substance):

E1p13. ∀x₅(Abs-infinite(x₅) → ¬Divisible(x₅))

To reconstruct Spinoza’s demonstrations of these propositions, we need to spell out the notion of divisibility. We take it that for Spinoza a thing is divisible if it can be divided into parts, where these parts are proper (E4p18s: “The whole is greater than its part”):

(PI) ∀x(Divisible(x) ≡ □∃y(Part(y, x) ∧ x ≠ y))

For related discussion of the indivisibility of substance and divisibility of modes, see also KV I, ii (1/26/9-16), Ep. 12 (IV/55/4-12), Ep. 35 (IV/182/1-7), and Ep. 36 (IV/184/24-30), some of which is quoted below.

Spinoza’s talk in E1p12d of a divided substance “losing the nature of substance” and “ceasing to be” suggests that he regards division as a temporal process—think of cutting a cake into slices. To more accurately model divisibility, we would need to add temporal structure to our Spinozian models and replace the atemporal metaphysical
With regard to substance in particular, Spinoza claims in E1p12d that the parts would be produced or “formed” from the substance:

\[(\Pi') \forall x ((\text{Substance}(x) \land \text{Divisible}(x)) \rightarrow \Diamond \exists y (\text{Part}(y, x) \land x \neq y \land x \sim y))\]

The divisibility of substance would therefore contradict E1p6 if the parts were themselves substances. In E1p12d, Spinoza also argues that the division of a single-faceted substance into other substances would lead to absurd results regarding the relationship between a whole and its parts. Intuitively, a whole must have something in common with each of its parts—if not, it isn’t clear how these can count as parts of this thing:

\[(\Pi II) \forall x \forall y (\text{Part}(y, x) \rightarrow \neg x \perp y)\]

However, the division of a substance into substance-parts would violate this principle by E1p2 and E1p5. It would also violate the mereological principle that wholes inhere in and are conceived through their parts (i.e., wholes depend on their parts in both an ontological and conceptual sense), which Spinoza endorses at a number of places in his writings (see for instance DPP 1p17, CM II 5 (I/258/16), Ep. 35 (IV/181/24), KV I, 2 (I/24/18-25/9)):  

\[(\Pi III) \forall x \forall y (\text{Part}(y, x) \rightarrow (x \leq y \land x \sqsubseteq y))\]

In the second horn of E1p12d, Spinoza also argues that the division of a substance into non-substance parts would result in the destruction of the original substance, contradicting E1p7. Spinoza elaborates on his conception of destruction in Ep. 36:

“A Being existing by its own sufficiency] cannot be divided into parts, either of the same or of a different nature, whether those of a different nature involve necessary existence or not. For, I said [in Ep. 35 (IV/182/2-3)], if the latter were the case, it could be destroyed [posset destruī], since to destroy a thing is to separate it into parts [rem destruere est illam in ejusmodi partes resolvere] of the same kind so that none of them expresses the nature of the whole. (IV/184/24-30)

Given this general conception of destruction as separation into parts none of which express the nature of the divided thing (see also E2d2), we can extract the premise that if a substance can be divided into parts none of which is a substance, then this substance can fail to exist (i.e., it can be destroyed):

\[(\Pi IV) \forall x ((\text{Substance}(x) \land \Diamond (\exists y (\text{Part}(y, x) \land x \neq y)) \land \forall z (\text{Part}(z, x) \rightarrow \neg \text{Substance}(z))) \rightarrow \Diamond \neg \text{Exists}(x))\]

possibility operator $\Diamond$ in PI with a future-directed possibility modal that considers whether the embedded sentence in its scope holds in some later time interval along a possible temporal trajectory.
Moving forward more carefully now, we reconstruct E1p12d as follows:

Proof. Suppose there is some $\bar{a}$ in $\Theta$ where $\text{Substance}(\bar{a})$ and $\text{Divisible}(\bar{a})$. By PI’ applied at the secondary sortal level, there is a metaphysically possible world $w \in W$ in which there exist $\bar{b}, \bar{c}, \ldots$ where $\text{Part}(\bar{b}, \bar{a}), \text{Part}(\bar{c}, \bar{a}), \ldots$, $\bar{a} \neq \bar{b}, \bar{a} \neq \bar{c}, \ldots$, and $\bar{a} \rightsquigarrow \bar{b}, \bar{a} \rightsquigarrow \bar{c}, \ldots$. Suppose that $\text{Substance}(\bar{b}), \text{Substance}(\bar{c}), \ldots$, that is, the parts “retain the nature of the substance”, so for instance $\Box \text{Infinite-in-kind}(\bar{b}), \Box \text{Infinite-in-kind}(\bar{c}), \ldots$ by E1p8, $\text{Causa-sui}(\bar{b}), \text{Causa-sui}(\bar{c}), \ldots$ by E1p7, and $\neg \text{Same-sort}(\bar{a}, \bar{b}), \neg \text{Same-sort}(\bar{a}, \bar{c}), \ldots$ by E1p5. Then the causal facts $\bar{a} \rightsquigarrow \bar{b}, \bar{a} \rightsquigarrow \bar{c}, \ldots$ contradict E1p6. Moreover, $\bar{a} \perp \bar{b}, \bar{a} \perp \bar{c}, \ldots$ by E1p2—contradicting PII.

And in case this wasn’t enough, the whole $\bar{a}$ inheres in only itself and is conceived through only itself by E1d4 and E1d3* (see also E1p10)—contradicting PIII. Next, suppose that $\neg \text{Substance}(\bar{b}), \neg \text{Substance}(\bar{c}), \ldots$, that is, the parts fail to retain the nature of substance. By PIV, $\Diamond \neg \exists \text{Sub}(\bar{a})$—contradicting E1p7. Because both forks of the proof lead to contradiction, $\neg \text{Divisible}(\bar{a})$.

The demonstration of E1p13 is similar, except that we need the analog of PIV concerning the destruction of God:

(PV) $\forall x_N((\text{Abs-infinite}(x_N) \land \Diamond (\exists y_N(\text{Part}(y_N, x_N) \land x_N \neq y_N)) \land \forall z_N(\text{Part}(z_N, x_N) \rightarrow \neg \text{Abs-infinite}(z_N)))) \rightarrow \Diamond \neg \exists \text{Sub}(x_N))$

In words: An absolutely infinite substance divisible into parts none of which is absolutely infinite can fail to exist.

Proof. Suppose there is $a_N$ in $\Theta$ where $\text{Abs-infinite}(a_N)$ and $\text{Divisible}(a_N)$. By PI applied at the $N$-level, there is a metaphysically possible world $w \in W$ in which there exist $b_N, c_N, \ldots$ where $\text{Part}(b_N, a_N), \text{Part}(c_N, a_N), \ldots$ and $a_N \neq b_N, a_N \neq c_N, \ldots$. Suppose that $\text{Abs-infinite}(b_N), \text{Abs-infinite}(c_N), \ldots$, that is, the parts “retain the nature of absolutely infinite substance”. For each secondary sort $s \in \{\text{Th}, \text{Ex}, \ldots\}$, $\pi_s(w)(a_N), \pi_s(w)(b_N), \pi_s(w)(c_N), \ldots$ are all defined. Given that $\pi_s(w)$ is a one-one injective function, we have multiple thinking substances, multiple extended substances, and multiple thinking substances, multiple extended substances, and

69While Spinoza rattles off these consequences of the supposition that the parts are substances, he doesn’t explicitly draw any contradictions from the inferences that the parts are infinite in their own kind and causes of themselves.

70It is implicitly assumed here that dividing a substance into other substances wouldn’t destroy it and therefore $\bar{a}$ continues to exist in $w$.

71Spinoza’s demonstration is rhetorically misleading as the fact that the parts are substances isn’t really needed here.

72In setting up his proof by cases, Spinoza does not consider the third option that some but not all of the parts retain the nature of the substance. This doesn’t make much of a difference, as his reasoning in the first subproof establishes that if any of the parts are substances, this contradicts E1p6, PII, and PIII.
so forth—contradicting E1p5. Suppose next that \( \neg \text{Abs-infinite}(b_R) \), \( \neg \text{Abs-infinite}(c_R) \), ... By PV, \( \Diamond \neg \exists(y_R) \) — contradicting E1p7*. So, \( \neg \text{Divisible}(a_R) \). □

Spinoza draws the following corollary from E1p12 and E1p13:

**Cor.:** From these [propositions] it follows that no substance, and consequently no corporeal substance, insofar as it is a substance, is divisible.

**E1p13c.** \( \forall x (\text{Substance}(x) \rightarrow \neg \text{Divisible}(x)) \)

(In particular: \( \forall x_{Ex} (\text{Substance}(x_{Ex}) \rightarrow \neg \text{Divisible}(x_{Ex})) \))

**Proof.** Consider \( a \) in @ where \( \text{Substance}(a) \). Either \( a \) is multifaceted or it is single-faceted. If multifaceted, then \( \text{Abs-infinite}(a) \) by the “Non-Gods are Chimeras” principle, so \( \neg \text{Divisible}(a) \) by E1p13. If single-faceted (for example, \( a \) might be a corporeal substance in \( D_{Ex}(@) \)), then \( \neg \text{Divisible}(a) \) by E1p12.

See E1p15s for further discussion and an alternative argument for the indivisibility of extended substance in particular.

Having already proven in E1p11 that God exists, Spinoza now proves in E1p14 that God is unique, thereby establishing his *substance monism*:

**P14:** Except God, no substance can be or be conceived.

While claims like E1d1 and E1a7 concern whether things can or cannot be conceived *as such and such* (as existing/not existing), E1p14 instead concerns whether things of a particular kind (viz., substances other than God) are conceivable *simpliciter*. Introducing the new conceivability predicate ‘*Conceivable*(t)’ to formalize *t is conceivable*, we translate this proposition as follows:

**E1p14.** \( \exists x_R (\text{God}(x_R) \land \neg \Sigma y_R (\text{Substance}(y_R) \land y_R \neq x_R \land (\Diamond \exists(y_R) \lor \text{Conceivable}(y_R)))) \)

In words: God exists and no substance other than God (in the possibilist sense) can exist or be conceived. While one could treat ‘*Conceivable*(t)’ as an independent predicate interpreted directly by the interpretation function of a model, we prefer to unpack it in terms of the already available sentential modal operator ‘\( \Diamond \)’. Let us now assume that if one

---

Crucially, E1p5 is applied to only single-faceted substances, so its use in proving the indivisibility of God doesn’t undermine our earlier Gueroult-like interpretation. Smith (2014) offers a similar take: “Spinoza sees the division of an absolutely infinite substance into substances of the same nature as the original as involving an infinite number of infringements of 1p5. It is appropriate for Spinoza to regard these as infringements of 1p5 (as interpreted by Gueroult), because the substances in question are single-attribute substances” (p. 670).

Spinoza cites E1p11, though E1p7* (used in E1p11d1) seems to be required.
can conceive anything at all about a thing \( a \) when attending to only its essence (i.e., its conceivability set \( C(a) \) is nonempty), then at the very least one can conceive this thing as being self-identical (i.e., the proposition that \( a = a \) lies in \( C(a) \)). We can thus regard \( \text{‘Conceivable}(t)’ \) as equivalent to \( \Diamond \, t = t \). In any case, none of this matters much for our reconstruction of E1p14d itself, which requires only that conceivable things cannot be the causes of their own nonexistence (i.e., Chimeras):

\[
\Pi x(\text{Conceivable}(x) \rightarrow x \not \leftrightarrow x)
\]

Proof. By E1p11, there is some \( a_\mathbb{R} \) in \( @ \) where \( \text{God}(a_\mathbb{R}) \) and \( \Box \text{Exists}(a_\mathbb{R}) \). Suppose there is some \( b_\mathbb{R} \in \mathcal{D}_\mathbb{R} \) such that \( \text{Substance}(b_\mathbb{R}) \) and \( a_\mathbb{R} \neq b_\mathbb{R} \). Suppose also that \( \Diamond \text{Exists}(b_\mathbb{R}) \)—that is, there is a metaphysically possible world \( w \in \mathcal{W} \) where \( b_\mathbb{R} \) exists. For some secondary sort \( s \in \{ \text{Th}, \text{Ex}, \ldots \} \), \( \pi_s(w)(b_\mathbb{R}) \) must be defined (this is built into the structure of projection functions). However, All sorts \( (a_\mathbb{R}) \) by E1d6, so \( \pi_s(w)(a_\mathbb{R}) \) is also defined and \( \pi_s(w)(a_\mathbb{R}) \neq \pi_s(w)(b_\mathbb{R}) \)—contradicting E1p5.\(^{75}\) Next, suppose that \( \text{Conceivable}(b_\mathbb{R}) \). By PI and the reasoning in E1p11d1 and E1p11d2, \( \Box b_\mathbb{R} \text{Exists}(b_\mathbb{R}) \) (and \( \neg \Diamond b_\mathbb{R} \neg \text{Exists}(b_\mathbb{R}) \) by E1d1b and E1d1h), contradicting that \( \neg \Diamond \text{Exists}(b_\mathbb{R}) \).\(^{76}\) So, neither \( \Diamond \text{Exists}(b_\mathbb{R}) \) nor \( \text{Conceivable}(b_\mathbb{R}) \).

Spinoza draws a couple of corollaries from E1p14:

**Cor 1.**: From this it follows most clearly, first, that God is unique, i.e. (by D6), that in Nature there is only one substance, and that it is absolutely infinite.

**E1p14c1.** \( \exists x_\mathbb{R}(\text{God}(x_\mathbb{R}) \land \forall y_\mathbb{R}(\text{Substance}(y_\mathbb{R}) \rightarrow y_\mathbb{R} = x_\mathbb{R})) \)

Proof. This follows immediately from E1p14 in the actual world \( @ \) given that \( @ \) is metaphysically possible. \[ \blacksquare \]

Now that Spinoza has finally established the uniqueness of God, we use the constant ‘\( \text{God}_R \)’ to refer to the unique witness of the existential quantifier in E1p14c1. Though E1p14c1 is stated at the primary \( R \)-level, the Projection Constraint in Spinozian models ensures that this result filters down to each of the secondary sortal domains—the projection of \( \text{God}_R \) into Extension (i.e., \( \text{God}_{\text{Ex}} \)) is the unique substance in \( \mathcal{D}_{\text{Ex}}(\@) \), the projection of \( \text{God}_R \) into Thought (i.e., \( \text{God}_{\text{Th}} \)) is the unique substance in \( \mathcal{D}_{\text{Th}}(\@) \), and so forth.

**Cor 2.**: It follows, second, that an extended thing and a thinking thing are either attributes of God, or (by A1) affections of God’s attributes.

\(^{75}\)Again, E1p5 applies only to single-faceted substances, so \textit{pace} Garrett (1990) its use in this context doesn’t undermine our Gueroult-like interpretation.

\(^{76}\)Spinoza’s reasoning in this second horn is pretty fast: “For if [a substance except God] could be conceived, it would have to be conceived as existing. But this (by the first part of this demonstration) is absurd.”
We reconstruct the part of this claim about Extension (and the case of Thought is exactly parallel):

**E1p14c2.** \( \forall x \exists y (\text{God}_N = p \ y \wedge (x_{\text{Ex}} = y \vee \text{Affection}(x_{\text{Ex}}, y))) \)

In words: For any extended thing, there is an attribute (projection) of God such that this extended thing is either this attribute or an affection of it.

**Proof.** Consider \( a_{\text{Ex}} \) in @. By the reasoning in E1p4d using E1a1, either \( \text{Substance}(a_{\text{Ex}}) \) or \( \text{Mode}(a_{\text{Ex}}) \). If \( \text{Substance}(a_{\text{Ex}}) \), then \( a_{\text{Ex}} = \pi_{\text{Ex}}(@)(\text{God}_N) \), where \( \pi_{\text{Ex}}(@)(\text{God}_N) \) is the unique extended substance (i.e., \( \text{God}_N \)) in \( D_{\text{Ex}}(@) \) by E1p14c1 and the Projection Constraint. So, \( \text{God}_N = p \ a_{\text{Ex}} \): the extended thing \( a_{\text{Ex}} \) is one of God’s attributes. If \( \text{Mode}(a_{\text{Ex}}) \), then \( \text{Affection}(a_{\text{Ex}}, b) \) for some \( b \) where \( \text{Substance}(b) \) by E1d5. By E1p14c1 and the Projection Constraint, \( \text{God}_N = p \ b \) (in fact, \( b \) must be \( \text{God}_N \)), so \( a_{\text{Ex}} \) is in this case an affection of one of God’s attributes. ■

In the final proposition considered in this paper, Spinoza proves that God grounds the existence and conception of all things:

**P15:** Whatever is, is in God, and nothing can be or be conceived without God.

We reconstruct this claim at the \( \aleph \)-level:

**E1p15.** \( \forall x_{\aleph}(x_{\aleph} \leq \text{God}_N) \wedge \neg \exists x_{\aleph}(\diamond \exists(x_{\aleph}) \wedge x_{\aleph} \not\leq \text{God}_N) \vee (\text{Conceivable}(x_{\aleph}) \wedge x_{\aleph} \not\subseteq \text{God}_N) \)

In words: Everything inheres in God and nothing can possibly exist but not inhere in God or be conceivable but not be conceived through God.

**Proof.** Consider \( a_{\aleph} \in D_{\aleph} \) such that \( \diamond \exists(x_{\aleph}) \) or \( \text{Conceivable}(a_{\aleph}) \). By the kind of reasoning in E1p4d using E1a1, either \( \text{Substance}(a_{\aleph}) \) or \( \text{Mode}(a_{\aleph}) \).\(^{77}\) If \( \text{Substance}(a_{\aleph}) \), then \( a_{\aleph} = \text{God}_N \) by E1p14, so \( a_{\aleph} \leq \text{God}_N \) by E1d3 at any metaphysically possible world \( w \in W \) (including @). Suppose next that \( \text{Mode}(a_{\aleph}) \). If \( a_{\aleph} \) exists in a metaphysically possible world \( w \in W \), then by E1d5, there exists \( b_{\aleph} \) in \( w \) such that \( \text{Substance}(b_{\aleph}) \) and \( a_{\aleph} \leq b_{\aleph} \). But \( b_{\aleph} = \text{God}_N \) by E1p14, so again \( a_{\aleph} \leq \text{God}_N \). Moreover, if \( \text{Conceivable}(a_{\aleph}) \), then by E1d5, there exists \( b_{\aleph} \) such that \( \text{Substance}(b_{\aleph}) \) and \( a_{\aleph} \subseteq b_{\aleph} \).\(^{78}\) But once more \( b_{\aleph} = \text{God}_N \) by E1p14, so \( a_{\aleph} \subseteq \text{God}_N \). ■

Fifteen propositions into the *Ethics*, Spinoza has already considerably limned the space of candidate models for his metaphysics. We know from E1p14 and E1p15 that in any model for the *Ethics* the \( \aleph \)-domain of @

\(^{77}\)The proof requires that conceivable things also be either substance or mode. One could make this explicit by adding another tacit premise or by generalizing the argument in E1p4d accordingly.

\(^{78}\)Assuming here that the definition of mode extends to conceivable things.
includes only God and its affections—that is, for all \( a_R \in D_R(\@) \), either \( a_R = \text{God}_R \) or Affection\( (a_R, \text{God}_R) \). We know from E1p7d and E1p13 that \( \text{God}_R \) is a causa sui \( (\text{Causa-sui}(\text{God}_R)) \) and indivisible \( (\neg \text{Divisible}(\text{God}_R)) \).

At the secondary level of attributes, the Projection Constraint ensures that each of God’s infinitely many attributes (i.e., extended substance \( \text{God}_{\text{Ex}} \), thinking substance \( \text{God}_{\text{Th}} \), and so on) is the unique substance in its respective sortal domain. Furthermore, we know from E1p7, E1p8, and Ep12 that each divine attribute exists necessarily by reason of its essence (e.g., \( \square \text{ Exists}(\text{God}_{\text{Ex}}) \)) is necessarily infinite in its own kind \( (\square \text{ Infinite-in-kind}(\text{God}_{\text{Ex}})) \) and is indivisible \( (\neg \text{Divisible}(\text{God}_{\text{Ex}})) \).

However, there is still plenty more limning to do. While Spinoza has already established that God exists in every metaphysically possible world, he hasn’t yet established that God’s essence fixes the full order of Nature (i.e., \( \bigcap \mathcal{E}(\text{God}_R) = \{\@\} \)). As things stand, a model for the Ethics through E1p15 can include multiple metaphysically possible ordi naturae. The current model theory also fails to capture Spinoza’s fine-grained ontology of finite and infinite modes. We leave these and further developments of the model theory for future installments of our project.

### 6 Previous Attempts

Our study is not the first attempt to represent the Ethics in a precise logical manner (or more logico, riffing on the Latin subtitle of the Ethics). As far as we know, the earliest attempt is found in George Boole’s Laws of Thought (1854), which contains a rudimentary formalization of what Boole considers “the most essential portions of the demonstrations of Spinoza” (i.e., the opening few pages of the Ethics) in an algebraic logic (pp. 211–218).

Boole sees Spinoza as effecting a number of parallel dichotomies of the universe of possible existents. This universe can be divided into things that are in themselves \( (x) \) or in another \( (x') \), into things that are conceived by themselves \( (y) \) or through another \( (y') \), into substances \( (z) \) or modes \( (z') \), into free things \( (f) \) or non-free things \( (f') \), and alternatively into self-caused things \( (e) \) or things caused by another \( (e') \). Using his symbolism, Boole suggests that E1d1, E1d3, E1d5, E1d7, E1a1, E1a2, E1a4, and E1a7 can be summarized in the following master series of equations:

\[
x = y = z = f = e = 1 - x' = 1 - y' = 1 - z' = 1 - f' = 1 - e'
\]

According to Boole, some of the core propositions of the Ethics are also

\[
\text{In Boole’s system, the symbols ‘x’, ‘y’, ‘z’, ... represent classes of individuals to which a particular name or description applies, ‘1’ is the full universe of discourse, and ‘−’ (“except”) is the sign for the negative mental operation whereby a whole is separated into parts. So, for example, ‘z = 1 – z’ tells us that the class of substances coincides with everything except the modes.}
\]
equations in this series. For instance, E1p6 amounts to \( z = 1 - e' \), and E1p7 to \( z = e \).\(^{80}\)

Boole’s algebraic repacking of the *Ethics* is inventive and charming but also fairly elementary, and it obviously falls well short of providing a faithful reconstruction of Spinoza’s actual argumentation in the text. A more sophisticated 20th-century attempt is provided by Charles Jarrett (1978), who presents Part One of the *Ethics* in a quantified modal logic. Jarrett’s system has some interesting features, including the availability of two different necessity operators: a strong operator conforming to S5 for representing the necessity of God’s existence, and a weaker operator conforming to T for representing the necessary existence of modes, which does not follow from their essences (Jarrett’s system also includes a necessity predicate). Like us, Jarrett regards the *Ethics* as a serious attempt to develop a consistent axiomatized metaphysics, and he tries to carefully extract the tacit premises needed for Spinoza’s demonstrations to go through.

While there is much to admire in Jarrett’s work, his formal system diverges from ours in many respects. Jarrett is concerned much more with proof theory than model theory, and he works with a standard Kripke semantics for modal logic—there are no primary and secondary sorts, projection functions, essence and conceivability functions, or other model-theoretic bells and whistles. Jarrett also conflates conceivability and possibility (see his treatment of E1d1 and E1a7), whereas we try to honor Spinoza’s delicate presentation of the two as parallel yet distinct notions. At a more fine-grained level, there are also numerous differences in our readings of the text. In fact, the majority of Jarrett’s translations of Spinoza’s definitions, axioms, and propositions differ significantly from our own. Furthermore, Jarrett merely formalizes Spinoza’s propositions and points out their likely sources, while we have more ambitiously tried to provide detailed reconstructions of Spinoza’s demonstrations of E1p1-E1p15 that faithfully trace his own reasoning.

For other more recent attempts to formalize the opening of the *Ethics*,

\(^{80}\)Of course, the vast majority of Spinoza’s propositions cannot be understood as equations in the master series. After putting forward this series as the logical heart of the *Ethics*, Boole oddly goes on to say that the *Ethics* is not in fact a fertile testing ground for his formal system of logic:

Though the “Ethics” of Spinoza, like a large portion of his other writings, is presented in the geometrical form, it does not afford a good praxis for the symbolical method of this work.... Reasoning which consists so largely of a play upon terms defined as equivalent, is not often met with; and it is rather on account of the interest attaching to the subject, than of the merits of the demonstrations, highly as by some they are esteemed, that I have devoted a few pages here to their exposition. (p. 216)

We don’t share Boole’s low opinion of Spinoza’s demonstrations, and rather think that he missed a good opportunity to better showcase his algebraic method.
see Blum & Malinovich (1993) and Sierra Márquez (2017). These also differ from our account in significant respects—Blum & Malinovich work with a non-modal logic, and Sierra Márquez omits attributes altogether in his formal implementation—but we leave detailed comparisons for another occasion.

7 Conclusion

While we believe our work is more precise than previous attempts to formalize the Ethics, we hope that this assertion will not remain true for long. We look forward to other scholars and teams joining us in an effort to provide the most exacting presentation—both philosophically and historically—of this unique work. From a bird’s eye perspective, we’d like to close by registering our admiration for Spinoza’s striking precision in drafting the Ethics. For the most part, Spinoza’s proofs of his propositions hang together remarkably well. Thoughout this paper, we have strived to stay close to the surface of the text. While we needed to introduce tacit premises along the way, these were fairly minimal (at least until E1p11d2, E1p12d, and E1p13d) and only a few carry controversial heavy-duty philosophical commitments (such as the use of the PSR in E1p7d, E1p11d1, and E1p11d2).81 So far, we believe the account of “Spinoza, the Logician” is a real success story.

Looking ahead, broadly speaking there are two directions along which to extend our study. First, one could push our formal reconstruction effort deeper into the Ethics by formalizing later parts of the geometrical core of the text. Second, one could work to provide more sophisticated implementations of Spinoza’s theories of essence, necessity, conception, conceivability, expression, involvement and the cluster of other notions already introduced in the fragment of Part One that we consider here. These research directions are not exclusive—as we’ve emphasized, an important route to sharpening our understanding of Spinozian concepts is to attend carefully to the role they play in his demonstrations.

Last but not least, we hope that our study will help undermine a common perception among philosophers and scholars of the history of philosophy that precise philosophical formalization is inconsistent with historical precision. Precise philosophy and precise history of philosophy needn’t come at the expense of one another, and in the current study we strived to achieve both kinds of precision by tracing the small nuances of the Ethics, its developmental history, and its proof strategies. As Spinoza suggests toward the conclusion of the Ethics, “Mentis enim oculi, quibus

81 Logical gaps, unstated tacit premises, and even implicit rules of inference are also common in Euclid’s Elements, which served as Spinoza’s model of deductive precision. For a recent study attempting to spell out tacit assumptions in the course of formalizing the Elements, see Avigad et al. (2009), especially pp. 701, 708–9, 763.
res videt, observatque, sunt ipsae demonstrationes [The eyes of the mind, by which it sees and observes things, are the demonstrations themselves]” (E5p23s). We want to conclude by marking the extraordinary vision of his mind at a time when the mental lenses of modern formal logic hadn’t yet been invented.

References


John Morrison. Spinoza on mind, body, and numerical identity. Unpublished manuscript.


